CHAPTER 18

Statistical Quality Control

LEARNING OBJECTIVES

This chapter presents basic concepts in quality control, with a particular emphasis on statistical quality control techniques, thereby enabling you to:

1. Explain the meaning of quality in business, compare the approaches to quality improvement by various quality gurus and movements, and compare different approaches to controlling the quality of a product, including benchmarking, just-in-time inventory systems, Six Sigma, lean manufacturing, reengineering, failure mode and effects analysis, poka-yoke, and quality circles
2. Compare various tools that identify, categorize, and solve problems in the quality improvement process, including flowcharts, Pareto analysis, cause-and-effect diagrams, control charts, check sheets, histograms, and scatter charts
3. Measure variation among manufactured items using various control charts, including $\bar{x}$ charts, *R* charts, *p* charts, and *c* charts

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Italy's Piaggio Makes a Comeback

Decision Dilemma

Piaggio, founded in Genoa, Italy, in 1884 by a 20-year-old young man named Rinaldo Piaggio, began as a luxury ship fitting company. Expanding on its services and products by the year 1900, the company was also producing rail carriages, luxury coaches, truck bodies, and trains. During the first World War, Piaggio began producing airplanes and sea-planes and then expanded capacity by purchasing a new plant in Pisa in 1917 and taking over a small plant in Pontedera (in Tuscany) in 1921, making the Pontedera plant the company's center for aeronautical production.

During World War II, the Pontedera plant was building state-of-the-art four-engine aircraft, but Allied planes destroyed the plant because of its military importance. With the Pontedera plant gone, the state of Italian roads a disaster, and the Italian economy in shambles, Enrico Piaggio (Rinaldo's son and now CEO) decided to focus the company's efforts on the personal mobility of the Italian people. Corradino D'Ascanio, Piaggio's ingenious aeronautical engineer who had designed, constructed, and flown the first modern helicopter, was commissioned to design a simple, sturdy, economical vehicle for people to get around in that was both comfortable and elegant. Drawing from his aeronautical background, D'Ascanio, who did not like motorcycles, developed a completely new vehicle that had a front fork, like an airplane, allowing for easy wheel changing, and was housed in a unibody steel chasis. It was not noisy or uncomfortable like a motorcycle, and the steel frame protected the rider from road dirt and debris. When Enrico Piaggio first saw the vehicle he said, "Sembra una Vespa!" ("It looks like a wasp!"), and as a result, the vehicle became known as the Vespa.

By the end of 1949, 35,000 Vespas had been produced, and in 10 more years, over 1 million had been manufactured. Featured in such films as *Roman Holiday, The Talented Mr. Ripley,* and *Alfie,* the Vespa became popular around the world and known as a global symbol of Italy and Italian design. In 1959, Piaggio came under control of the powerful Agnelli family, owners of the car-maker Fiat SpA, and for the next two decades, the scooter maker flourished. However, during the latter half of the twentieth century, revolving-door management and millions of euros wasted on ill-conceived expansion plans left the company with crushing debt and vulnerable to competition from companies in the Pacific Rim. Losing money and market share, Piaggio was caught up in a downward spiral of increasing debt, bad quality, and inability to meet market demand. As the twenty-first century arrived, the company's status was looking bleak until the year 2003, when Italian industrialist Roberto Colaninno bought the company. Implementing a series of strategic moves and quality initiatives, Colaninno turned around the fortunes of Piaggio, now the fourth largest manufacturer of scooters and motorcycles in the world, producing more than 600,000 vehicles annually. In a recent year, Piaggio had a revenue of $2.53 billion and a net income of almost $90 million.

Managerial and Statistical Questions

1. Was the decline of Piaggio driven by poor quality? If so, how?
2. What quality initiatives did Colaninno implement at Piaggio that helped turn the company around?
3. Were company workers consulted about ways to improve the product and the process?

Sources: Piaggio Vespa Web site: http://www.vespausa.com/company/history.cfm Gabriel Kahn, "Vespa's Builder Scoots Back to Profitability," *Wall Street Journal*, June 5, 2006, B1; Piaggio.com, 2009.

In the past three decades, institutions around the world have invested millions of dollars in improving quality, and in some cases, corporate cultures have been changed through the implementation of new quality philosophies. Much has been written and spoken about quality, and a great deal of research has been conducted on the effectiveness of various quality approaches. In order to study and explore the myriad of quality theories, approaches, tools, and concepts, it is important to first understand what is quality.

One major stumbling block to studying and implementing quality improvement methodologies is that quality means different things to different people. If you asked commuters whether their automobiles have quality, the response would vary according to each individual's perspective. One person's view of a quality automobile is one that goes 75,000 miles without needing any major repair work. Other people perceive automobile quality as comfortable seats and extra electronic gadgetry. These people look for "bells and whistles" along with form-fitting, cushy seats in a quality car. Still other automobile consumers define automobile quality as the presence of numerous safety features.

In this chapter, we examine various definitions of quality and discuss some of the main concepts of quality and quality control. We explore some techniques for analyzing processes. In addition, we learn how to construct and interpret control charts.

18.1 INTRODUCTION TO QUALITY CONTROL

There are almost as many definitions of quality as there are people and products. However, one definition that captures the spirit of most quality efforts in the business world is that **quality** is *when a product delivers what is stipulated for it in its specifications.* From this point of view, quality is when the producer delivers what has been specified in the product description, as agreed upon by both buyer and seller. Philip B. Crosby, a well-known expert on quality, has said that "quality is conformance to requirements."* The product requirements must be met by the producer to ensure quality. This notion of quality is similar to the one based on specifications. Armand V. Feigenbaum, a well-known quality authority, says in his book *Total Quality Control* that "quality is a customer determination" as opposed to management's determination or a designer's determination.† He states that this determination is based on the customer's experience with the product or service and that it is always a moving target.

David A. Garvin, author of *Managing Quality,* claims that there are at least five types of quality: transcendent, product, user, manufacturing based, and value.‡ **Transcendent quality** *implies that a product has an "innate excellence."* It has *"uncompromising standards and high achievement."* Garvin says that this definition offers little practical guidance to business people. **Product quality** *is measurable in the product.* Consumers perceive differences in products, *and quality products have more attributes.* For example, a personal computer with more memory has more quality. Tires with more tread have more quality.

User quality means that the *quality of a product is determined by the consumer* and is in the "eye of the beholder." One problem with user-based quality is that because there are widely varying individual preferences, there can be a plethora of views of quality for a given product or service. **Manufacturing-based quality** has to do with engineering and manufacturing practices. Once specifications are determined, *quality is measured by the manufacturer's ability to target the requirements consistently with little variability*. Most manufacturing-based definitions of quality have to do with "conformance to requirements." **Value quality** is defined in costs and prices. From a certain point-of-view, value quality is based on cost-benefit analysis; that is, by how much did the benefit of the good or service outweigh the cost? Did the customer get his or her money's worth?

What Is Quality Control?

How does a company know whether it is producing a quality product? One way is to practice quality control. **Quality control** (sometimes referred to as quality assurance) is *the collection of strategies, techniques, and actions taken by an organization to assure itself that it is producing a quality product.*

From this point of view, quality control begins with product planning and design, where attributes of the product or service are determined and specified, and continues through product production or service operation until feedback from the final consumer is looped backed through the institution for product improvement. It is implied that all

*Philip B. Crosby, *Quality Without Tears*. New York: McGraw-Hill, 1984.

†Armand V. Feigenbaum, *Total Quality Control,* 3rd ed. New York: McGraw-Hill, 1991.

‡David A. Garvin, *Managing Quality.* New York: The Free Press, 1988.

departments, workers, processes, and suppliers are in some way responsible for producing a quality product or service.

Quality control can be undertaken in two distinct ways: after-process control and in-process control. **After-process quality control** involves *inspecting the attributes of a finished product to determine whether the product is acceptable, is in need of rework, or is to be rejected and scrapped.* The after-process quality-control method was the leading quality-control technique for U.S. manufacturers for several decades until the 1980s. The after-process method emphasizes weeding out defective products before they reach the consumer. The problem with this method is that it does not generate information that can correct in-process problems or raw materials problems nor does it generate much information about how to improve quality. Two main outcomes of the after-process methodology are (1) reporting the number of defects produced during a specific period of time and (2) screening defective products from consumers. Because U.S. companies dominated world markets in many areas for several decades during and after World War II, their managers had little interest in changing from the after-process method.

However, as Japan, other Asian nations, and Western European countries began to compete strongly with the United States in the world market in the late 1970s and 1980s, U.S. companies began to reexamine quality-control methods. As a result, many U.S. companies, following the example of Japanese and European manufacturers, developed quality-control programs based on in-process control. **In-process quality control** *techniques measure product attributes at various intervals throughout the manufacturing process in an effort to pinpoint problem areas.* This information enables quality-control personnel in conjunction with production personnel to make corrections in operations as products are being made. This intervention in turn opens the door to opportunities for improving the process and the product.

Total Quality Management

W. Edwards Deming, who has been referred to as the "father of the quality movement," advocated that the achievement of quality is an organic phenomenon that begins with top managers' commitment and extends all the way to suppliers on one side and consumers on the other. Deming believed that quality control is a long-term total company effort. The effort called for by Deming is **total quality management** (**TQM**). Total quality management involves all members of the organization—from the CEO to the line worker—in improving quality. In addition, the goals and objectives of the organization come under the purview of quality control and can be measured in quality terms. Suppliers, raw materials, worker training, and opportunity for workers to make improvements all are part of total quality management. The antithesis of total quality management is when a company gives a quality-control department total responsibility for improving product quality.

Deming presented a cause-and-effect explanation of the impact of total quality management on a company. This idea has become known as the Deming chain reaction.* The chain reaction begins with improving quality. Improving quality will decrease costs because of less reworking, fewer mistakes, fewer delays and snags, and better use of machine time and materials. From the reduced costs comes an improvement in productivity because

$$\text{Productivity} = \frac{\text{Output}}{\text{Input}}$$

A reduction of costs generates more output for less input and, hence, increases productivity. As productivity improves, a company is more able to capture the market with better quality and lower prices. This capability enables a company to stay in business and provide more jobs. As a note of caution, while Deming advocated that improved quality results in a byproduct of lower costs through efficiencies gained by streamlining and reducing waste, some managers have used it as an excuse to lay off workers in an effort to save money.

*W. Edwards Deming, *Out of the Crisis*. Cambridge: Massachusetts Institute of Technology Center for Advanced Engineering Study, 1986.

It is likely that Deming would have argued that such cost-cutting actually reduces quality and productivity due to an increase in operational errors, errors of omission, and a lack of attention to detail by a reduced staff that is overworked, understaffed, and stressed.

Deming's 14 Points

Deming listed 14 points which, if followed, can lead to improved total quality management, and they are as follows*:

1. Create constancy of purpose for improvement of product and service.
2. Adopt the new philosophy.
3. Cease dependence on mass inspection.
4. End the practice of awarding business on price tag alone.
5. Improve constantly and forever the system of production and service.
6. Institute training.
7. Institute leadership.
8. Drive out fear.
9. Break down barriers between staff areas.
10. Eliminate slogans.
11. Eliminate numerical quotas.
12. Remove barriers to pride of workmanship.
13. Institute a vigorous program of education and retraining.
14. Take action to accomplish the transformation.

The first point indicates the need to seek constant improvement in process, innovation, design, and technique. The second point suggests that to truly make changes, a new, positive point of view must be taken; in other words, the viewpoint that poor quality is acceptable must be changed. The third point is a call for change from after-process inspection to in-process inspection. Deming pointed out that after-process inspection has nothing to do with improving the product or the service. The fourth point indicates that a company should be careful in awarding contracts to suppliers and vendors. Purchasers should look more for quality and reliability in a supplier than for just low price. Deming called for long-term supplier relationships in which the company and supplier agree on quality standards.

Point 5 conveys the message that quality is not a one-time activity. Management and labor should be constantly on the lookout for ways to improve the product. Institute training, the sixth point, implies that training is an essential element in total quality management. Workers need to learn how to do their jobs correctly and learn techniques that will result in higher quality. Point 7, institute leadership, is a call for a new management based on showing, doing, and supporting rather than ordering and punishing. The eighth point results in establishing a "safe" work environment, where workers feel free to share ideas and make suggestions without the threat of punitive measures. Point 9, breaking down barriers, emphasizes reducing competition and conflicts between departments and groups. It is a call for more of a team approach—the notion that "we're all in this together."

Deming did not believe that slogans help affect quality products, as stated in point 10. Quality control is not a movement of slogans. Point 11 indicates that quotas do not help companies make quality products. In fact, pressure to make quotas can result in inefficiencies, errors, and lack of quality. Point 12 says that managers must find ways to make it easier for workers to produce quality products; faulty equipment and poor-quality supplies do not allow workers to take pride in what is produced. Point 13 calls for total reeducation and training within a company about new methods and how to more effectively do one's job. Point 14 implies that rhetoric is not the answer; a call for action is necessary in order to institute change and promote higher quality.

*Mary Walton, *The Deming Management Method.* New York: Perigee Books, 1986.

Quality Gurus

Some other important and well-known quality gurus include Joseph Juran, Philip Crosby, Armand Feigenbaum, Kaoru Ishikawa, and Genichi Taguchi. Joseph Juran, a contemporary of Deming, also assisted Japanese leaders in the 1950s in implementing quality concepts and tools so that they could produce products that would be attractive to world markets. Juran was particularly well known for his "Juran Trilogy," which included Quality Planning, Quality Control, and Quality Improvement. Philip Crosby, author of the popular book, *Quality Is Free*, developed a zero-defects program to reduce defects in missile production in the United States in the late 1950s and early 1960s, and later he established a Quality College. Crosby bases his approach to quality on four basic tenets that he refers to as "Absolutes": 1.) Quality means conformance to requirements, 2.) Defect prevention is the only acceptable approach, 3.) Zero defects is the only performance standard, and 4.) The cost of quality is the only measurement of quality. Armand Feigenbaum has been a worldwide leader in quality management for over a half century. Feigenbaum published his widely-read text, *Total Quality Control*, in 1951 under the title, *Quality Control: Principles, Practice, and Administration*. While W. Edwards Deming is often associated with total quality management, it was Feigenbaum who actually coined the term "total quality control." He originated the concept of "cost of quality" as a means of quantifying the benefits of a TQM approach, and he popularized the term "hidden factory," which describes the part of plant capacity wasted due to poor quality. Kaoru Ishikawa, a student of both Deming and Juran, is probably the most well-known figure in the Japanese quality movement. He has been credited with originating the concept of "quality circle" and championed what is now seen as Japan's "company-wide" approach to quality. Ishikawa placed an emphasis on data measurement and using statistical techniques in improving quality. He is known for developing the "cause-and-effect" or "fishbone" diagram that is sometimes referred to as the "Ishikawa" diagram. Genichi Taguchi, an important figure in the Japanese quality movement, wrote a two-volume book on experimental designs that has been widely used in the quality-improvement efforts. In the 1970s, Taguchi developed the concept of the Quality Loss Function and refined a set of cost-saving quality-improvement techniques that later became known as Taguchi Methods.

Six Sigma

Currently, a popular approach to total quality management is Six Sigma. **Six Sigma** is a quality movement, a methodology, and a measurement. As a quality movement, Six Sigma is a major player throughout the world in both the manufacturing and service industries. As a methodology, it is used to evaluate the capability of a process to perform defect-free, where a defect is defined as anything that results in customer dissatisfaction. Six Sigma is customer focused and has the potential to achieve exponential quality improvement through the reduction of variation in system processes. Under the Six Sigma methodology, quality-improvement projects are carefully defined so that they can be successfully completed within a relatively short time frame. Financials are applied to each completed project so that management can estimate how much the project saves the institution. On each project, intense study is used to determine root cause. In the end, a metric known as a "sigma level" can be assigned to represent the level of quality that has been attained, and this is the measurement aspect of Six Sigma.

The Six Sigma approach to quality is said to have begun in 1987 with Bill Smith, a reliability engineer at Motorola.* However, Six Sigma took off as a significant quality movement in the mid-1990s when Jack Welch, CEO of General Electric, "went nuts about Six Sigma and launched it," calling it the most ambitious task the company had ever taken on.† "Six Sigma has taken the corporate world by storm and represents the thrusts of numerous efforts in manufacturing and service organizations to improve products, services, and

*James R. Evans and William M. Lindsay, *The Management and Control of Quality*, 5th ed. Cincinnati: South-Western Publishing, 2002.

†Jack Welch, *Jack: Straight from the Gut*. New York: Warner Books, 2001, pp. 329–330.

processes."* Six Sigma has been around for almost 20 years and has shown a sustained impact on quality improvement within a variety of companies in many industries. Six Sigma is derived from a previous quality scheme in which a process was considered to be producing quality results if $\pm 3\sigma$ or 99.74% of the products or attributes were within specification. (Note: The standard normal distribution table, Table A.5, produces an area of .4987 for a z score of 3. Doubling that and converting to a percentage yields 99.74%, which is the portion of a normal distribution that falls within $\mu \pm 3\sigma$.) Six Sigma methodology requires that $\pm 6\sigma$ of the product be within specification. The goal of Six Sigma methodology is to have 99.99966% of the product or attributes be within specification, or no more than .00034% = .0000034 out of specification. This means that no more than 3.4 of the product or attributes per million can be defective. Essentially, it calls for the process to approach a defect-free status.

Why Six Sigma? Several reasons highlight the importance of adoption of a Six Sigma philosophy. First, in some industries the three sigma philosophy is simply unacceptable. For example, the three sigma goal of having 99.74% of the product or attribute be in specification in the prescription drug industry implies that it is acceptable to have .26% incorrectly filled prescriptions, or 2,600 out of every million prescriptions filled. In the airline industry, the three sigma goal implies that it is acceptable to have 2,600 unsatisfactory landings by commercial aircraft out of every million landings. In contrast, a Six Sigma approach would require that there be no more than 3.4 incorrectly filled prescriptions or 3.4 unsatisfactory landings per million, with a goal of approaching zero.

A second reason for adopting a Six Sigma approach is that it forces companies that adopt it to work much harder and more quickly to discover and reduce sources of variation in processes. It "raises the bar" of the quality goals of a firm, causing the company to place even more emphasis on continuous quality improvement. A third reason is that Six Sigma dedication to quality may be required to attain world-class status and be a top competitor in the international market.

Six Sigma contains a formalized problem-solving approach called the DMAIC process (*D*efine, *M*easure, *A*nalyze, *I*mprove, and *C*ontrol). At the beginning of each Six Sigma project, the project team carefully identifies the problem—not just the symptoms—at the Define stage. The scope of the project is limited so that it can be completed within 4 to 6 months. At the Measure stage, there is a heavy emphasis on metrics and measurement of current operating systems along with identifying variables and targeting data collection. During the Analyze stage, there is a focus on analyzing data and collected information in an effort to determine what is occurring and uncovering root causes of problems. At the fourth stage, Improve, the project team generates ideas for solving problems and improving performance. Lastly, at the fifth stage, Control, there is a focus on putting into motion those tools, standards, etc., that are needed to maintain the new quality that has been achieved through the Six Sigma project.

Another important aspect of Six Sigma as a methodology and a quality movement is the strong focus on the customer that is often referred to as CTQ or Critical to Quality. Maintaining a customer focus is vital to every stage of a Six Sigma project, keeping in mind that there are both internal and external customers. Six Sigma project team members work on things that are important to the customer and do not spend time on things that could be improved but are not important to the customer (not CTQ).

Under Six Sigma, most members of an organization are supposed to have at least some training in the methodology. Employees with minimal exposure to Six Sigma—perhaps only an introductory lecture—might be designated as "yellow belts" (named after the belt system in karate). Organizational members who have more extensive training and serve part-time on Six Sigma teams are designated as "green belts." Fully trained employees having over 150 hours of training in Six Sigma, usually including at least one Six Sigma project, are called "black belts." Black belts work full time within an organization usually directing several Six Sigma projects simultaneously. Master black belts are "experts" in Six Sigma. They have advanced training in statistical techniques and other Six Sigma tools and methods, and they work in the organization developing teams, training black belts, and providing technical direction.

*James R. Evans and William M. Lindsay, *An Introduction to Six Sigma & Process Improvement*. Cincinnati Thomson South-Western Publishing Company, 2005, p. 4.

Design for Six Sigma

Companies using Six Sigma discovered that some processes, outcomes, and services, often designed before the Six Sigma era, contained so many flaws and problems that even the in-depth, root analysis of the Six Sigma methodology could not solve some quality issues, and thus, a complete redesign was necessary. In fact, history has shown that most processes can only achieve about a 5.0 sigma status with quality improvement and to actually achieve 6.0 sigma status, organizations often need to *design* for 6.0 sigma; that is, because of constraints or limitations built-in by its original design, there may be a ceiling on how much a process or operation can be improved. Design for Six Sigma (DFSS), an off-shoot from Six Sigma, is a quality scheme that places an emphasis on designing the product or process right the first time, thereby allowing organizations the opportunity to reach even higher sigma levels through Six Sigma.

Lean Manufacturing

Lean manufacturing is a quality-management philosophy that focuses on the reduction of wastes and the elimination of unnecessary steps in an operation or process. Whereas the tenets of lean manufacturing have existed in successful manufacturing circles for over a century, the Toyota Production System is generally credited with developing the notion of lean manufacturing as it exists today. Lean manufacturing requires a disciplined attitude in seeking out and eliminating waste in all areas of business including supplier networks, customer relations, organization management, and design. Proponents of lean manufacturing claim it brings about an evaluation of the entire organization and restructures processes to reduce wasteful activities.

In particular, lean manufacturing focuses on seven wastes:

1. overproduction
2. waiting time
3. transportation
4. processing
5. inventory
6. motion
7. scrap.

Overproduction can include making more than is needed or making it earlier than is needed. Waiting includes products waiting on the next production step or people waiting for work to do. Transportation waste can include moving products further than is minimally required, and inventory waste can include having more inventory than is minimally required at any point in the process, including end-product. Processing waste is doing more work than the customer values or needs, and motion waste is having people move around unnecessarily or wasting motion in performing their production or operation functions.*

Some advocates of lean manufacturing claim that even if a process or service is operating at a Six Sigma level, it does not necessarily follow that the process or service is lean. That is, the quality of the process or service can be quite high, but there can still be waste in the system. Some critics of Six Sigma say that just improving the quality does not necessarily reduce the time that it takes to perform the process. With this in mind, a new approach to quality management has been developed by combining the investigative, variation reduction aspects of Six Sigma with the emphasis on increased efficiency of lean manufacturing, resulting in what some refer to as Lean Six Sigma.

Some Important Quality Concepts

Of the several widely used techniques in quality control, six in particular warrant discussion: benchmarking, just-in-time inventory systems, reengineering, Failure Mode and Effects Analysis, poka-yoke, and quality circles and Six Sigma teams.

*Adapted from Wikipedia, the free encyclopedia, at: http://en.wikipedia.org/wiki/Lean_manufacturing.

Benchmarking

One practice used by U.S. companies to improve quality is benchmarking. **Benchmarking** is *a method in which a company attempts to develop and establish total quality management from product to process by examining and emulating the "best practices" and techniques used in its industry.* The ultimate objective of benchmarking is to use a positive, proactive process to make changes that will affect superior performance. The process of benchmarking involves studying competitors and learning from the best in the industry.

One of the American pioneers in what is called "competitive benchmarking" was Xerox. Xerox was struggling to hold on to its market share against foreign competition. At one point, other companies could sell a machine for what it cost Xerox to make a machine. Xerox set out to find out why. The company instituted a benchmarking process in which the internal workings and features of competing machines were studied in depth. Xerox attempted to emulate and learn from the best of these features in developing its own products. In time, benchmarking was so successful within the company that top managers included benchmarking as a major corporate effort.*

Just-in-Time Inventory Systems

Another technique used to improve quality control is the just-in-time system for inventory, which focuses on raw materials, subparts, and suppliers. Ideally, a **just-in-time inventory system** means that *no extra raw materials or inventory of parts for production are stored.* Necessary supplies and parts needed for production arrive "just in time." The advantage of this system is that holding costs, personnel, and space needed to manage inventory are reduced. Even within the production process, as subparts are assembled and merged, the just-in-time philosophy can be applied to smooth the process and eliminate bottlenecks.

A production facility is unlikely to become 100% just-in-time. One of the residual effects of installing a just-in-time system throughout the production process is that, as the inventory "fat" is trimmed from the production process, the pressure on the system to produce often discloses problems previously undetected. For example, one subpart being made on two machines may not be produced in enough quantity to supply the next step. Installation of the just-in-time system shows that this station is a bottleneck. The company might choose to add another machine to produce more subparts, change the production schedule, or develop another strategy. As the bottleneck is loosened and the problem is corrected, other areas of weakness may emerge. Thus, the residual effect of a just-in-time inventory system can be the opportunity for production and operations managers to work their way methodically through a maze of previously unidentified problems that would not normally be recognized.

A just-in-time inventory system typically changes the relationship between supplier and producer. Most companies using this system have fewer suppliers than they did before installing the system. The tendency is for manufacturers to give suppliers longer contracts under the just-in-time system. However, the suppliers are expected to produce raw materials and subparts to a specified quality and to deliver the goods as near to just in time as possible. Just-in-time suppliers may even build production or warehouse facilities next to the producer's. In the just-in-time system, the suppliers become part of total quality management.

Just-in-time (JIT) as a management philosophy has come to mean "eliminating manufacturing wastes by producing only the right amount and combination of parts at the right place at the right time,"[†] that is, JIT now generally signifies production with a minimum of waste.[‡] The goal of JIT is to minimize "non–value-adding operations" and "nonmoving inventory" in a production process or operation.[§] In this sense, some view JIT, also known as "lean production,"[§] as the forerunner of what is now referred to as "lean manufacturing." While some of the basic elements of JIT were used by Toyota in the 1950s, most historians give credit to Taiichi Ohno of Toyota for developing JIT in the 1970s, and Ohno is often referred to as the father of JIT.[§]

*Robert C. Camp, *Benchmarking.* Milwaukee, WI: Quality Press, ASQC, 1989.

[†]Quoted from the Web site for SEMICON FAREAST at: http://www.semiconfareast.com/jit.htm.

[‡]From http://www.ifm.eng.cam. ac.uk/dstools/process/jit.html.

[§]http://www.semiconfareast.com/jit.htm.

There are basic elements that underscore the just-in-time philosophy, and some of these include*:

1. Leveling the loads on work centers to smooth the flow of goods or services
2. Reducing or even eliminating setup times
3. Reducing lot sizes
4. Reducing lead times
5. Conducting preventive maintenance on machines and equipment to ensure that they work perfectly when needed
6. Having a flexible work force
7. Requiring supplier quality assurance and implement a zero-defects quality program
8. Improving, eliminating, or reducing anything that does not add value to the product
9. Striving for simplicity
10. Making each worker responsible for the quality of his or her output

By leveling loads at workstations throughout the process, bottlenecks are reduced or eliminated, and there is a greater chance that goods/services will flow smoothly through the process.

Reengineering

A more radical approach to improving quality is reengineering. Whereas total quality approaches like Deming's 14 points call for continuous improvement, **reengineering** is *the complete redesigning of the core business process in a company.* It involves innovation and is often a complete departure from the company's usual way of doing business.

Reengineering is not a fine-tuning of the present process nor is it mere downsizing of a company. Reengineering starts with a blank sheet of paper and an idea about where the company would like to be in the future. Without considering the present limitations or constraints of the company, the reengineering process works backward from where the company wants to be in the future and then attempts to determine what it would take to get there. From this information, the company cuts or adds, reshapes, or redesigns itself to achieve the new goal. In other words, the reengineering approach involves determining what the company would be like if it could start from scratch and then redesigning the process to make it work that way.

Reengineering affects almost every functional area of the company, including information systems, financial reporting systems, the manufacturing environment, suppliers, shipping, and maintenance. Reengineering is usually painful and difficult for a company. Companies that have been most successful in implementing reengineering are those that faced big shifts in the nature of competition and that required major changes to stay in business.

Some recommendations to consider in implementing reengineering in a company are to (1) get the strategy straight first, (2) lead from the top, (3) create a sense of urgency, (4) design from the outside in, (5) manage the firm's consultant, and (6) combine top-down and bottom-up initiatives. Getting the strategy straight is crucial because the strategy drives the changes. The company must determine what business it wants to be in and how to make money in it. The company's strategy determines its operations.

The focus of reengineering is outside the company; the process begins with the customer. Current operations may have some merit, but time is spent determining the need of the marketplace and how to meet that need.

A company need not necessarily reengineer its entire operation. A specific process, such as billing, production, distribution, etc., can be reengineered. For example, a mortgage loan company may completely rethink and restructure the process for which a loan applicant gets approved. In healthcare, a hospital might radically redesign its admissions procedure to significantly reduce admissions time and stress for both the patient and the admissions officer. An integral part of the reengineering effort is to question basic assumptions and

*From http://www.ifm.eng.cam.ac.uk/dstools/process/jit.html and http://personal.ashland.edu/~rjacobs/m503jit.html.

traditions, rethink the way business has been done, and reinvent the process so that significant improvements can be made.

Failure Mode and Effects Analysis

Failure Mode and Effects Analysis, or FMEA, is a systematic way for identifying the effects of a potential product or process failure and includes methodology for eliminating or reducing the chance of a failure occurring.* It is used for analyzing potential reliability problems early in the development cycle where it is easier to take actions to overcome these issues, thereby enhancing reliability through design. A crucial step in the FMEA analysis is anticipating what might go wrong with a product, and while anticipating every failure mode is not possible, a development team should formulate an extensive list of potential failure modes when implementing a FMEA analysis. FMEA was developed in the 1960s in the aerospace industry and is now used extensively in the automotive industry. It was developed as a preventative mechanism and is most useful when it is implemented before a product or process is released rather than later. Because FMEA helps engineers identify potential product or process failures, they can minimize the likelihood of those failures, track and manage risks associated with the product or process, and ensure that failures that do occur will not injure or seriously impact the customer. In this way, they can help provide reliable, safe, and customer pleasing products and processes.‡

The first step in a FMEA analysis starts with the selection of the process or product to be reviewed and its function. Most FMEA projects involve processes or products that are candidates for a high risk of error. Initially, investigators determine which uses of the product or process fall inside the intended use and which fall outside, since product failure often leads to litigation.† Next, they assemble a team made up of people who understand or use the product or process regularly.‡ Using a block diagram showing the major components and how they are related or a flowchart, the team searches to identify locations, types, and severities of failure modes. Such failure modes might include cracking, fatigue, corrosion, electrical short, spoilage, and others. After identifying failure modes, a criticality index or risk priority number is assigned to each. The Risk Priority Number, or RPN, is calculated by multiplying the Severity of the problem times the Probability that it will occur times the Detection rating. Severity has to do with the seriousness of the problem brought about by the failure. Occurrence is the frequency with which the failure happens. Detection is the likelihood that a failure can be discovered or noticed. Using the RPN, the failure mode items can be prioritized for action. The items with the highest scores receive the highest priorities and are acted on first. Such action could include inspections, testing, implementing quality control procedures, using different components or materials, limiting the operating range, redesigning the item, performing preventative maintenance, and including back-up systems or redundancy.† After action is taken, someone or a group is assigned the responsibility for implementing the actions by a target completion date. After actions are taken, a FMEA reassessment is taken for possible RPN revision.

Most FMEA efforts include the use of an FMEA Worksheet or FMEA Data Collection and Analysis Form. Shown below is one such example.‡

PROCESS:								
Function/ Task	Failure Mode	Effect	Severity Score	Occurrence Score	Detection Score	RPN = $S \cdot O \cdot D$	Recommended Action	Target Date

*Pat Hammett, University of Michigan, "Failure Mode and Effects Analysis," PowerPoint presentation available at: http://www.fmeainfocentre.com/.

†Kenneth Crow, DRM Associates, "Failure Modes and Effects Analysis (FMEA)." Document found at: http://www.npd-solutions.com/fmea/html

‡Donald E. Lighter and Douglas C. Fair, *Quality Management in Health Care: Principles and Methods,* 2nd ed. Boston: Jones and Bartlett 2004, p. 85.

Poka-Yoke

Another common quality concept that can be used in continuous improvement is Poka-Yoke, which means "mistake proofing." Poka-Yoke, pronounced (POH-kah YOH-kay) and developed by Japanese industrial engineer Shigeo Shingo in the early 1960s, uses devices, methods, or inspections in order to avoid machine error or simple human error. There are two main types of poka-yokes: 1.) prevention-based poka-yokes and 2.) detection-based poka-yokes. Prevention-based poka-yokes are mechanisms that sense that an error is about to occur and send some sort of signal of the occurrence or halt the process. Detection poka-yokes identify when a defect has occurred and stop the process so that the defect is not "built-in" to the product or service and sent downstream.

In contrast to Deming, who believed that poor quality is generally not the fault of the worker (but rather equipment, training, materials, etc.), Shingo believed that "The causes of defects lie in worker errors, and defects are the results of neglecting those errors. It follows that mistakes will not turn into defects if worker errors are discovered and eliminated beforehand."* As an example, suppose a worker is assembling a device that contains two push-buttons. A spring must be placed under each push-button in order for it to work. If either spring is not put in place, the associated push-button will not work, and an error has been committed. If the worker does an on-the-spot inspection by looking at the device or by testing it, then the cost of fixing the device (rework) is minimal, both in terms of the time to fix it and the time lost due to inspection. If, on the other hand, the error is not identified and the device goes on down the assembly line and is incorporated as a part into a product, the cost of rework, scrap, repair, or warrantee claim can become quite high. A simple poka-yoke solution might be that the worker first counts two springs out of the supply bin and places them both in a small dish before each assembly. If the worker is not paying attention, is daydreaming, or is forgetful, he/she merely needs to look at the dish to easily see if there is a leftover spring before sending the device on to the next station. Some other examples of poka-yoke include machines with limit switches connected to warning lights that go on if an operator improperly positions a part on a machine, computer programs displaying warning messages if a file is being closed but has not been saved, plugs on the back of the computer tower that have different sizes and/or shapes along with color codes to prevent the connection of a plug into the wrong hole, electric hedge trimmers that force the user to hold down two "switches" before the trimmers will work to make it harder for the user to accidentally cut themselves, and a plate, which is only supposed to be screwed down in one position (orientation), that has screw holes in non-symmetrical positions so that the holes only line up for mounting if the plate is in the proper position.

Shingo believed that most mistakes can be prevented if people make the effort to identify when and where errors happen and take corrective actions. Simple poka-yoke mechanisms, such as a device, a switch, a guide pin, a procedure, or a visual inspection, can go a long way in preventing errors that result in defects and thereby reduce productivity.

Quality Circles and Six Sigma Teams

In years past, the traditional business approach to decision making in the United States allowed managers to decide what was best for the company and act upon that decision. In the past decade or so, the U.S. business culture underwent major changes as total quality management was adopted. One aspect of total quality management is team building. **Team building** occurs *when a group of employees are organized as an entity to undertake management tasks and perform other functions such as organizing, developing, and overseeing projects.*

The result of team building is that more workers take over managerial responsibilities. Fewer lines of demarcation separate workers from managers and union from nonunion. Workers are invited to work on a par with managers to remove obstacles that prevent a company from delivering a quality product. The old "us and them" point of view is being

*Shigeo Shingo, *Zero Quality Control: Source Inspection and the Poka-Yoke System*. University Park, IL: Productivity Press, 1986, p. 50.

STATISTICS IN BUSINESS TODAY

Six Sigma Focus at GE

GE's focus on quality began late in the 1980s with a movement called Work-Out which reduced company bureaucracy, opened their culture to new ideas, and helped create a learning environment that eventually led to Six Sigma. In the mid-1990s, General Electric established a goal of attaining Six Sigma quality by the year 2000. By 1999, GE had already invested more than $1 billion in their quality effort. Today at General Electric, Six Sigma defines the way they do business. They continue to strive for greater quality by following a Six Sigma philosophy. In their push for Six Sigma status, they engaged more than 5,000 employees in Six Sigma methodology in more than 25,000 completed projects.

GE's Six Sigma approach is data driven and customer focused. All of their employees are trained in the techniques, strategy, and statistical tools of Six Sigma. The focus of Six Sigma at GE is on reducing process variation and increasing process capability. The benefits delivered by this process include reduced cycle times, accelerated product designs, consistent efforts to eliminate variation, and increased probabilities of meeting customer requirements. The adoption of Six Sigma resulted in a culture in which quality thinking is embedded at every level in every operation throughout the company.

Why has GE pursued a Six Sigma philosophy? GE discovered that their customers were demanding better quality, and their employees thought they could be doing a better job. Their peer competitors such as Motorola, Texas Instruments, and Allied Signal had proven that following a disciplined, rigorous approach to quality significantly improved customer service and resulted in greater productivity. Internal process defects had been limiting GE's ability to achieve growth objectives. With increased globalization and information access, GE believes that products and services continually change the way their customers do business, and the highly competitive worldwide marketplace leaves no room for error in designing, producing, and delivering products. Six Sigma provides the philosophy and approach needed to meet these goals.

The GE people point out the difference between three sigma and Six Sigma: With three sigma, there are 1.5 misspelled words per page in a book in a small library; with Six Sigma there is 1 misspelled word in all the books in a small library. In a post office with three sigma, there are 20,000 lost articles of mail per hour; with Six Sigma, there are only 7 per hour. They also claim that Six Sigma can improve your golf score. If you played 100 rounds of golf per year, under a two sigma philosophy you would miss six putts per round. Under three sigma, you would miss one putt per round. Under Six Sigma, you would miss only one putt every 163 years!

Source: Adapted from General Electric, "Quality: GE's Evolution Towards Quality, What Is Six Sigma?" and "Achieving Quality for the Customer," available at http://www.ge.com/commitment/quality.htm; "Tip 'Six Sigma' Quality," accessed (formerly available) at http://trailers.ge.com/getip/news/june.html, and "What We Do," accessed (formerly available) at http://www.crd.ge.com/whatwedo/sixsigma.html.

replaced by a cooperative relationship between managers and workers in reaching common goals under team building.

One particular type of team that was introduced to U.S. companies by the Japanese is the quality circle. A **quality circle** is *a small group of workers,* usually from the same department or work area, and their supervisor, *who meet regularly to consider quality issues.* The size of the group ranges from 4 to 15 members, and they meet as often as once a week.* The meetings are usually on company time and members of the circle are compensated. The supervisor may be the leader of the circle, but the members of the group determine the agenda and reach their own conclusions.

The Six Sigma approach to quality makes use of teams of various "belts" to work on Six Sigma projects. Such Six Sigma project teams are usually led by a black belt, who is a company employee, works full-time on Six Sigma projects, and has received extensive training in Six Sigma methodology. Virtually all Six Sigma team members possess at least a green belt and have at least some job-released time to work on the project. Somewhat in contrast to the traditional quality circle teams that come from a particular department or group, Six Sigma teams often include members from various functional groups in the organization, some of whom may come from different levels of the company. For example, it is not uncommon for hospital Six Sigma team membership to include physicians, nurses, pharmacists, technicians, administrators, and others in an attempt to uncover root cause, take targeted measurements, and brainstorm possible solutions.

**The American Heritage Dictionary of the English Language,* 3rd ed. Boston: Houghton Mifflin Company, 1992.

18.2 PROCESS ANALYSIS

Much of what transpires in the business world involves processes. A **process** is *"a series of actions, changes, or functions that bring about a result."* Processes usually involve the manufacturing, production, assembling, or development of some output from given input. Generally, in a meaningful system, value is added to the input as part of the process. In the area of production, processes are often the main focus of decision makers. Production processes abound in the chemical, steel, automotive, appliance, computer, furniture, and clothing manufacture industries, as well as many others. Production layouts vary, but it is not difficult to picture an assembly line with its raw materials, parts, and supplies being processed into a finished product that becomes worth more than the sum of the parts and materials that went into it. However, processes are not limited to the area of production. Virtually all other areas of business involve processes. The processing of a check from the moment it is used for a purchase, through the financial institution, and back to the user is one example. The hiring of new employees by a human resources department involves a process that might begin with a job description and end with the training of a new employee. Many different processes occur within healthcare facilities. One process involves the flow of a patient from check-in at a hospital through an operation to recovery and release. Meanwhile, the dietary and foods department prepares food and delivers it to various points in the hospital as part of another process. The patient's paperwork follows still another process, and central supply processes medical supplies from the vendor to the floor supervisor.

There are many tools that have been developed over the years to assist managers and workers in identifying, categorizing, and solving problems in the continuous quality-improvement process. Among these are the seven basic tools of quality developed by Kaoru Ishikawa in the 1960s.* Ishikawa believed that 95% of all quality-related problems could be solved using those basic tools,† which are sometimes referred to as the "seven old tools."‡ The seven basic tools are as follows:

1. Flowchart or process map
2. Pareto chart
3. Cause-and-effect diagram (Ishikawa or fishbone chart)
4. Control chart
5. Check sheet or checklist
6. Histogram
7. Scatter chart or scatter diagram

Flowcharts

One of the first activities that should take place in process analysis is the flowcharting of the process from beginning to end. A **flowchart** is *a schematic representation of all the activities and interactions that occur in a process.* It includes decision points, activities, input/output, start/stop, and a flowline. Figure 18.1 displays some of the symbols used in flowcharting.

The parallelogram represents input into the process or output from the process. In the case of the dietary/foods department at the hospital, the input includes uncooked food, utensils, plates, containers, and liquids. The output is the prepared meal delivered to the patient's room. The processing symbol is a rectangle that represents an activity. For the dietary/foods department, that activity could include cooking carrots or loading food carts. The decision symbol, a diamond, is used at points in the process where decisions are made that can result in different pathways. In some hospitals, the dietary/foods department supports a hospital cafeteria as well as patient meals. At some point in the process, the decision

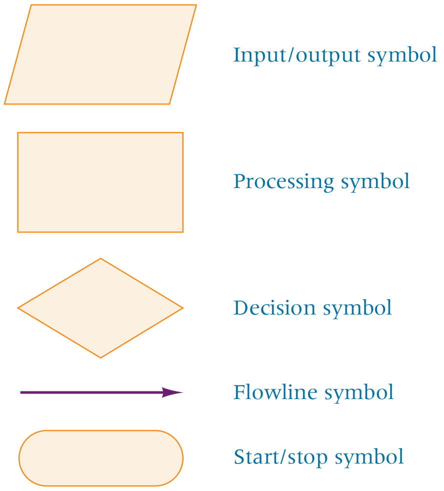
*Jason Paster, April 2, 2001. Internet source found at: http://www.freequality.org/sites/www_freequality_org/Documents/knowledge/basicseventools.pdf.

†MPR Associates, Inc., Web site: http://www.devicelink.com/mddi/archive/98/04/012.html.

‡Nancy R. Tague. *The Quality Toolbox*, 2nd ed. Milwaukee, WI: ASQ Press, 2004, p. 15.

FIGURE 18.1

Flowchart Symbols



Source: G. A. Silver and J. B. Silver, *Introduction to Systems Analysis*. Englewood Cliffs, N.J.: Prentice Hall, 1976, 142–147.

must be made as to whether the food is destined for a patient room or the cafeteria. The cafeteria food may follow a general menu whereas patient food may have to be individualized for particular health conditions. The arrow is the flowline symbol designating to the flowchart user the sequence of activities of the process. The flowline in the hospital food example would follow the pathway of the food from raw ingredients (vegetables, meat, flour, etc.) to the delivered product in patient rooms or in the cafeteria. The elongated oval represents the starting and stopping points in the process.

Particularly in nonmanufacturing settings, it is common that no one maps out the complete flow of sequential stages of various processes in a business. For example, one NASA subcontractor was responsible for processing the paperwork for change items on space projects. Change requests would begin at NASA and be sent to the subcontractor's building. The requests would be processed there and returned to NASA in about 14 days. Exactly what happened to the paperwork during the 2-week period? As part of a quality effort, NASA asked the contractor to study the process. No one had taken a hard look at where the paperwork went, how long it sat on various people's desks, and how many different people handled it. The contractor soon became involved in process analysis.

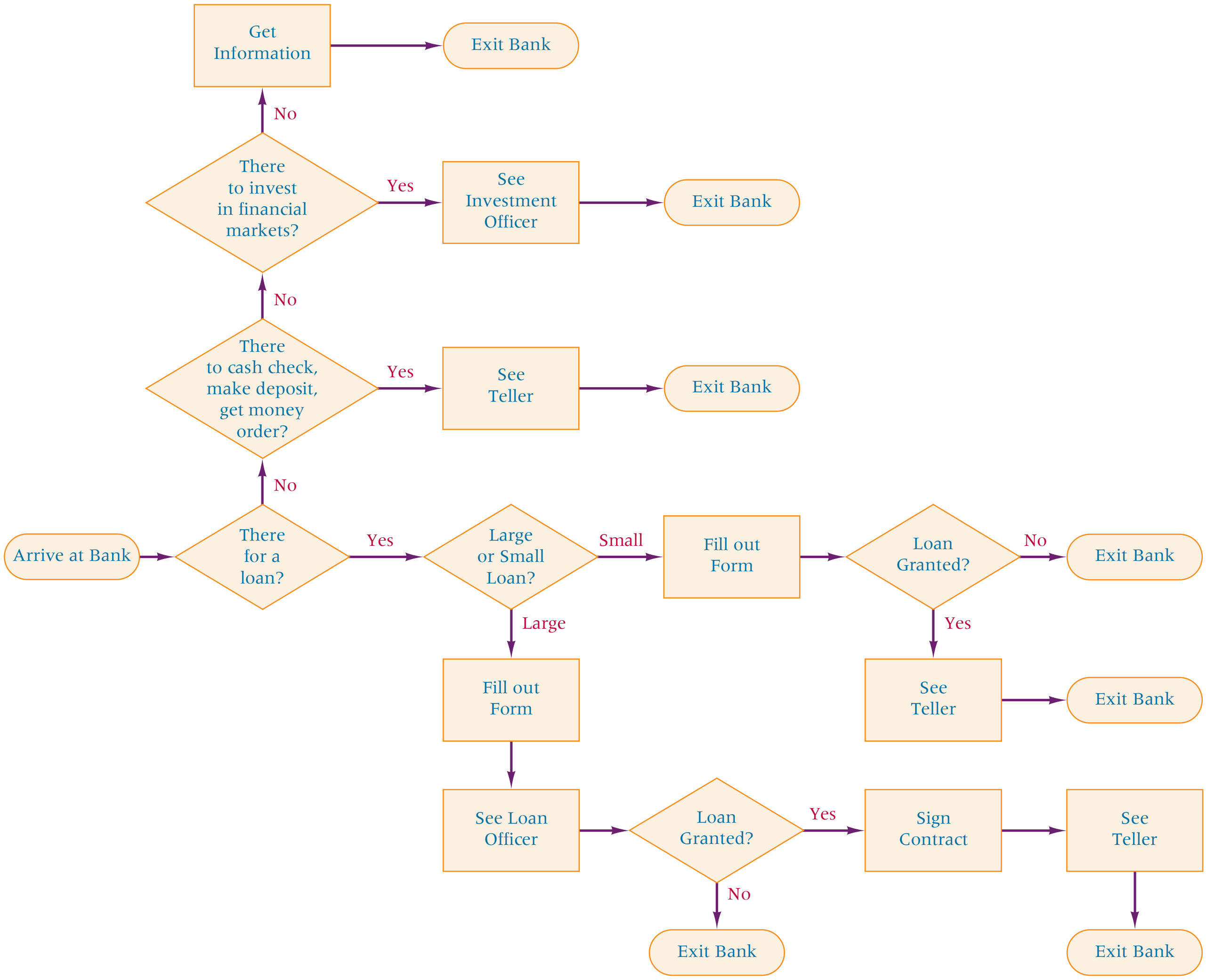
As an example, suppose we want to flowchart the process of obtaining a home improvement loan of $10,000 from a bank. The process begins with the customer entering the bank. The flow takes the customer to a receptionist, who poses a decision dilemma. For what purpose has the customer come to the bank? Is it to get information, to cash a check, to deposit money, to buy a money order, to get a loan, or to invest money? Because we are charting the loan process, we follow the flowline to the loan department. The customer arrives in the loan department and is met by another receptionist who asks what type and size of loan the person needs. For small personal loans, the customer is given a form to submit for loan consideration with no need to see a loan officer. For larger loans, such as the home improvement loan, the customer is given a form to fill out and is assigned to see a loan officer. The small personal loans are evaluated and the customer is given a response immediately. If the answer is yes, word of the decision is conveyed to a teller who cuts a check for the customer. For larger loans, the customer is interviewed by a loan officer, who then makes a decision. If the answer is yes, a contract is drawn up and signed. The customer is then sent to a teller who has the check for the loan. Figure 18.2 provides a possible flowchart for this scenario.

Pareto Analysis

Once the process has been mapped by such techniques as the flowchart, procedures for identifying bottlenecks and problem causes can begin. One technique for displaying

FIGURE 18.2

Flowchart of Loan Process



problem causes is Pareto analysis. **Pareto analysis** is *a quantitative tallying of the number and types of defects that occur with a product or service.* Analysts use this tally to produce *a vertical bar chart that displays the most common types of defects, ranked in order of occurrence from left to right.* The bar chart is called a **Pareto chart**. Pareto charts are presented and explained in greater detail in Section 2.3 of Chapter 2. Figure 18.3 contains a Minitab Pareto chart depicting various potential sources of medication error in a hospital. Figure 18.4 redisplays Figure 2.10, which depicts the possible causes of motor problems.

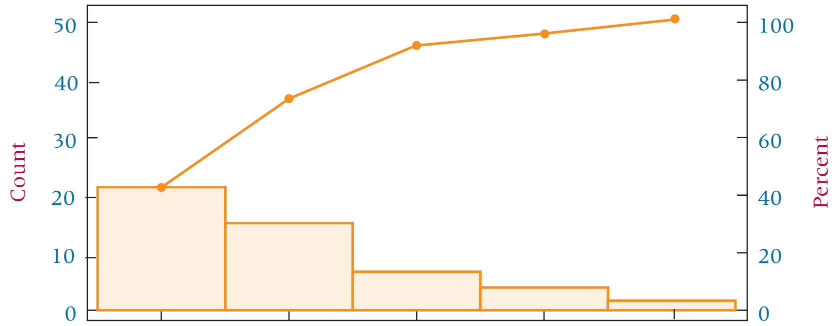
Cause-and-Effect (Fishbone) Diagrams

Another tool for identifying problem causes is the **cause-and-effect diagram**, sometimes referred to as **fishbone**, or **Ishikawa, diagram**. This diagram was developed by Kaoru Ishikawa in the 1940s as a way to *display possible causes of a problem and the interrelationships among the causes.* The causes can be uncovered through brainstorming, investigating, surveying, observing, and other information-gathering techniques.

The name *fishbone diagram* comes from the shape of the diagram, which looks like a fish skeleton with the problem at the head of the fish and possible causes flaring out on both sides of the main "bone." Subcauses can be included along each "fishbone."

FIGURE 18.3

Pareto Chart of Medication Errors in a Hospital



Defect	Wrong Medicine Administered	Improperly Stored	Misread Handwriting	Wrong Prescription Written	Improperly Filled
Count	22	16	8	4	2
Percent	42.3	30.8	15.4	7.7	3.8
Cum %	42.3	73.1	88.5	96.2	100.0

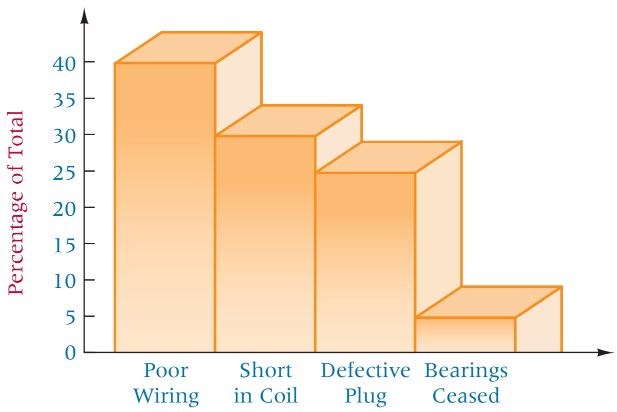
Suppose officials at the company producing the electric motor want to construct a fishbone diagram for the poor wiring problem shown as the major problem in Figure 18.4. Some of the possible causes of poor wiring might be raw materials, equipment, workers, or methods. Some possible raw material causes might be vendor problems (and their source of materials), transportation damage, or damage during storage (inventory). Possible causes of equipment failure might be out-of-date equipment, equipment that is out of adjustment, poor maintenance of equipment, or lack of effective tools. Poor wiring might also be the result of worker error, which can include lack of training or improper training, poor attitude, or excessive absenteeism that results in lack of consistency. Methods causes can include poor wiring schemes and inefficient plant layouts. Figure 18.5 presents a Minitab fishbone diagram of this problem and its possible causes.

Control Charts

A fourth diagnostic technique that has worldwide acceptance is the control chart. According to Armand V. Feigenbaum, a renowned expert on control charts, a **control chart** is *a graphical method for evaluating whether a process is or is not in a "state of statistical control."**

FIGURE 18.4

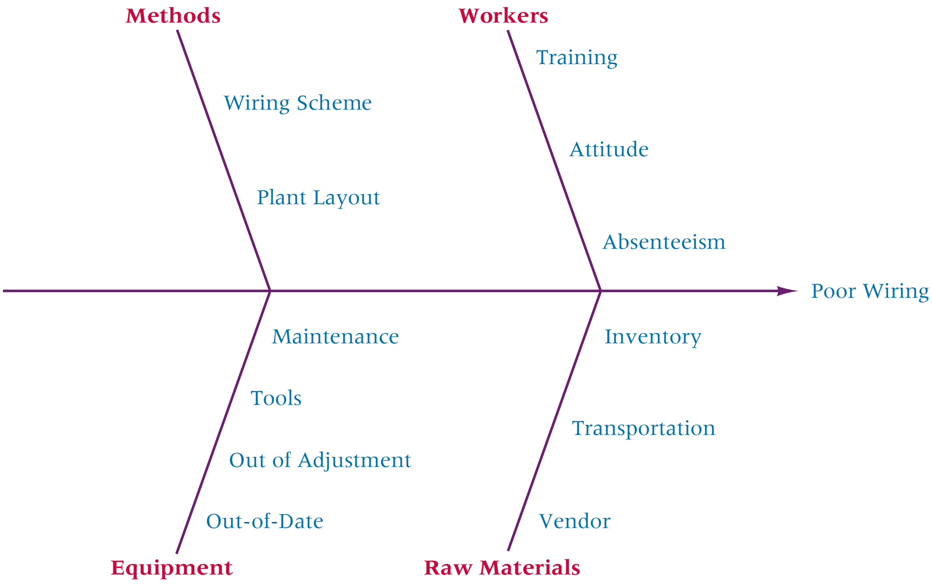
Pareto Chart for Electric Motor Problems



*Armand V. Feigenbaum, *Total Quality Control*. New York, McGraw-Hill, 1991.

FIGURE 18.5

Minitab Cause-and-Effect Diagram for Electric Motor Problems



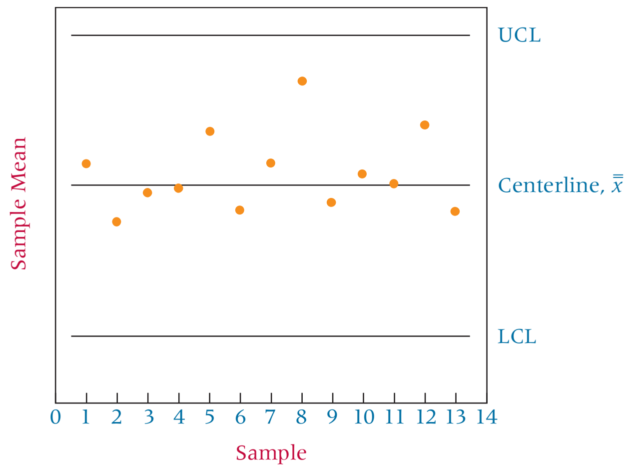
Several kinds of control charts are used. Figure 18.6 is an $\bar{x}$ control chart. In the next section, we will explore control charts in more detail.

Check Sheets or Checklists

Check sheets, sometimes called checklists, come in a variety of forms but usually display the frequency of outcomes for some quality-related event or activity under study or being observed by using some type of matrix in which an observer records in a category each outcome from the event or activity. Most check sheets are simple forms consisting of multiple categories and columns for recording tallies and are used for collecting data in a logical format and helping organize data by category. Some advantages of using check sheets are they are simple to use, they convert raw data into useful information, and the results may be interpreted on the form directly without additional processing.* Check sheets are especially

FIGURE 18.6

$\bar{x}$ Control Chart



*James R. Evans and William M. Lindsay, *The Management and Control of Quality*, 5th ed. Cincinnati South-Western Publishing, 2002, p. 609.

useful in tracking problems and causes of problems and for providing hard evidence that supports fact rather than having to rely on opinion. They show how many times each particular item or value occurs, and their information is increasingly helpful as more data are collected. One of the side-effects of using check sheets is that the person using them becomes very aware of the data being captured and can often see patterns building.*

In constructing a check sheet, there are several things to consider. First, decide what problem, activity, or event is being studied. Once the problem to study has been determined, it can be helpful to involve "process" workers in developing the check sheet categories. Once the problem has been identified and categories have been determined, the form can be designed. Check sheets should be user-friendly with all columns labeled clearly, and a format should be created that gives you the most information with the least amount of effort.†

Shown below is an example of a check sheet that could be used to determine why patients in a hospital do not consume a meal within 1 hour of its delivery.‡ Assume that a team of nurses, aides, dietary personnel, patients, and others was assembled to identify possible causes for this problem.

Date:
Floor:
Shift: **Meal Consumption within 1 hour of delivery**

CHECKSHEET	MON	TUE	WED	THU	FRI	SAT	SUN	TOTAL
Menu Incorrect			1			1		2
Diet Order Changed	1			1			1	3
Wrong Order Delivered		1	1		1		1	4
Patient Asleep	11	111	1	11	111	11	111	16
Patient Out of Room	1111	1111	11	11	1	111	1	17
Doctor Making Rounds		1	111	11	1	1	1	9
Patient Not Hungry	1		11	1			1	5
Plate Warmer Broken	11	11	11	11			1	9
Nursing Unavailable	11	1	1111	111	11	1	1	14
TOTAL	**12**	**12**	**16**	**13**	**8**	**8**	**10**	**79**

From this check sheet it can be gleaned that the top three reasons for meals not being consumed within 1 hour of delivery are: patient asleep, patient out of the room, and nursing unavailable.

Histogram

Another important tool for quality improvement is the histogram. A histogram is a type of vertical bar chart used to depict a frequency distribution and was presented in Section 2.2 of Chapter 2 of this textbook. Most computer software packages, such as Excel, can produce a histogram from raw data. The histogram is a very useful tool in getting an initial overview of the data.

Scatter Chart or Scatter Diagram

Many times in the implementation of quality improvement techniques and in root-cause analysis, it is important to explore the relationship between two numerical variables. One graphical mechanism for examining the relationship between two variables is the scatter chart, sometimes referred to as a scatter diagram. A scatter chart is formally defined to be a two-dimensional graph plot of pairs of points from two numerical variables. Scatter

*Web site: http://syque.com/improvement/Check%20Sheet.htm.

†De La Salle University Web site at: http://quality.dlsu.edu.ph/tools/check_sheet.html.

‡Adapted from example shown at: http://mot.vuse.vanderbilt.edu/mt322/Check.htm.

charts are presented and discussed as scatter plots in Section 2.4 of Chapter 2 of this textbook. Often a scatter chart can aid the quality investigator in determining if there is a relationship between two variables; and if there is a relationship, what the direction of the relationship is. One note of caution about using the scatter chart as a tool is that even though there may appear to be a relationship between two variables (and indeed there may be a relationship between two variables), that does not mean that one variable causes the other variable to change or "drives" the other variable.

18.2 PROBLEMS

18.1 For each of the following scenarios, sketch the process with activities and decision points. Construct a flowchart by using the symbols depicted in Figure 18.1.

a. A customer enters the office of an insurance agent wanting to purchase auto insurance and leaves the office with a paid policy in hand.

b. A truckload of men's shirts enters a warehouse at the main distribution center for a men's retail clothing company that has four stores in that area. The shirts are on the racks inside the stores ready for sale.

c. A couple enters a restaurant to enjoy dinner out. An hour and a half later, they leave, satisfied, having paid their bill. Construct the flowchart from the restaurant's point of view.

18.2 An airline company uses a central telephone bank and a semiautomated telephone process to take reservations. It has been receiving an unusually high number of customer complaints about its reservation system. The company conducted a survey of customers, asking them whether they had encountered any of the following problems in making reservations: busy signal, disconnection, poor connection, too long a wait to talk to someone, could not get through to an agent, connected with the wrong person. Suppose a survey of 744 complaining customers resulted in the following frequency tally.

Number of Complaints	Complaints
184	Too long a wait
10	Transferred to the wrong person
85	Could not get through to an agent
37	Got disconnected
420	Busy signal
8	Poor connection

Construct a Pareto chart from this information to display the various problems encountered in making reservations.

18.3 A bank has just sent out a monthly reconciliation statement to a customer with an error in the person's monthly income. Brainstorm to determine some possible causes of this error. Try to think of some possible reasons for the causes. Construct a fishbone diagram to display your results.

18.3 CONTROL CHARTS

Control charts have been in existence for nearly 70 years. Walter A. Shewhart is credited with developing control charts at Bell Laboratories in the 1920s. Shewhart and others, including one of his understudies at Bell Laboratories, W. Edwards Deming, were able to apply control charts to industrial processes as a tool to assist in controlling variation. The use of control charts in the United States failed to gain momentum after World War II because the success of U.S. manufacturers in the world market reduced the apparent need for such a tool. As the Japanese and other international manufacturers became more competitive by using such tools, the control chart increased in popularity in the United States.

Control charts are easy to use and understand. Often it is the line workers who record and plot product measurements on the charts. In more automated settings, sensors record chart values and send them into an information system, which compiles the charts. Control charts are used mainly to monitor product variation. The charts enable operators, technicians, and managers to see when a process gets out of control, which in turn improves quality and increases productivity.

Variation

If no variations occurred between manufactured items, control charts would be pointless. However, variation occurs for virtually any product or service. Variation can occur among units within a lot and can occur between lots. Among the reasons for product variation are differences in raw materials, differences in workers, differences in machines, changes in the environment, and wear and tear of machinery. Small variations can be caused by unnoticeable events, such as a passing truck that creates vibrations or dust that affects machine operation. Variations need to be measured, recorded, and studied so that out-of-control conditions can be identified and corrections can be made in the process.

Types of Control Charts

The two general types of control charts are (1) control charts for measurements and (2) control charts for attribute compliance. In this section, we discuss two types of control charts for measurements, $\bar{x}$ charts and R charts. We also discuss two types of control charts for attribute compliance, p charts and c charts.

Each control chart has a **centerline**, an **upper control limit (UCL)**, and a **lower control limit (LCL)**. Data are recorded on the control chart, and the chart is examined for disturbing patterns or for data points that indicate a process is out of control. Once a process is determined to be out of control, measures can be taken to correct the problem causing the deviation.

$\bar{x}$ Chart

An $\bar{x}$ **chart** is *a graph of sample means computed for a series of small random samples over a period of time.* The means are average measurements of some product characteristic. For example, the measurement could be the volume of fluid in a liter of rubbing alcohol, the thickness of a piece of sheet metal, or the size of a hole in a plastic part. These sample means are plotted on a graph that contains a centerline and upper and lower control limits (UCL and LCL).

$\bar{x}$ charts can be made from standards or without standards.* Companies sometimes have smoothed their process to the point where they have standard centerlines and control limits for a product. These standards are usually used when a company is producing products that have been made for some time and in situations where managers have little interest in monitoring the overall measure of central tendency for the product. In this text, we will study only situations in which no standard is given. It is fairly common to compute $\bar{x}$ charts without existing standards—especially if a company is producing a new product, is closely monitoring proposed standards, or expects a change in the process. Many firms want to monitor the standards, so they recompute the standards for each chart. In the no-standards situation, the standards (such as mean and standard deviation) are estimated by using the sample data.

The centerline for an $\bar{x}$ chart is the average of the sample means, $\bar{\bar{x}}$. The $\bar{x}$ chart has an upper control limit (UCL) that is three standard deviations of means above the centerline ($+3\sigma_{\bar{x}}$). The lower boundary of the $\bar{x}$ chart, called the lower control limit (LCL), is three standard deviations of means below the centerline ($-3\sigma_{\bar{x}}$). Recall the empirical rule presented in Chapter 3 stating that if data are normally distributed, approximately 99.7% of all values will be within three standard deviations of the mean. Because the shape of the sampling distribution of $\bar{x}$ is normal for large sample sizes regardless of the population shape, the empirical rule applies. However, because small samples are often used, an approximation of the three standard deviations of means is used to determine

*Armand V. Feigenbaum, *Total Quality Control*. New York: McGraw-Hill, 1991.

UCL and LCL. This approximation can be made using either sample ranges or sample standard deviations. For small sample sizes ($n \le 15$ is acceptable, but $n \le 10$ is preferred), a weighted value of the average range is a good approximation of the three-standard-deviation distance to UCL and LCL. The range is easy to compute (difference of extreme values) and is particularly useful when a wide array of nontechnical workers are involved in control chart computations. When sample sizes are larger, a weighted average of the sample standard deviations ($\bar{s}$) is a good estimate of the three standard deviations of means. The drawback of using the sample standard deviation is that it must always be computed, whereas the sample range can often be determined at a glance. Most control charts are constructed with small sample sizes; therefore, the range is more widely used in constructing control charts.

Table A.15 contains the weights applied to the average sample range or the average sample standard deviation to compute upper and lower control limits. The value of A_2 is used for ranges and the value of A_3 is used for standard deviations. The following steps are used to produce an $\bar{x}$ chart.

1. Decide on the quality to be measured.
2. Determine a sample size.
3. Gather 20 to 30 samples.
4. Compute the sample average, $\bar{x}$, for each sample.
5. Compute the sample range, R, for each sample.
6. Determine the average sample mean for all samples, $\bar{\bar{x}}$, as

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{k}$$

   where k is the number of samples.
7. Determine the average sample range for all samples, $\bar{R}$, as

$$\bar{R} = \frac{\Sigma R}{k}$$

   or determine the average sample standard deviation for all samples, $\bar{s}$, as

$$\bar{s} = \frac{\Sigma s}{k}$$

8. Using the size of the samples, n_i, determine the value of A_2 if using the range and A_3 if using standard deviations.
9. Construct the centerline, the upper control limit, and the lower control limit. For ranges:

$\bar{\bar{x}}$ is the centerline

$\bar{\bar{x}} + A_2\bar{R}$ is the UCL

$\bar{\bar{x}} - A_2\bar{R}$ is the LCL

For standard deviations:

$\bar{\bar{x}}$ is the centerline

$\bar{\bar{x}} + A_3\bar{s}$ is the UCL

$\bar{\bar{x}} - A_3\bar{s}$ is the LCL

DEMONSTRATION PROBLEM 18.1

A manufacturing facility produces bearings. The diameter specified for the bearings is 5 millimeters. Every 10 minutes, six bearings are sampled and their diameters are measured and recorded. Twenty of these samples of six bearings are gathered. Use the resulting data and construct an $\bar{x}$ chart.

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
5.13	4.96	5.21	5.02	5.12
4.92	4.98	4.87	5.09	5.08
5.01	4.95	5.02	4.99	5.09
4.88	4.96	5.08	5.02	5.13
5.05	5.01	5.12	5.03	5.06
4.97	4.89	5.04	5.01	5.13
Sample 6	**Sample 7**	**Sample 8**	**Sample 9**	**Sample 10**
4.98	4.99	4.96	4.96	5.03
5.02	5.00	5.01	5.00	4.99
4.97	5.00	5.02	4.91	4.96
4.99	5.02	5.05	4.87	5.14
4.98	5.01	5.04	4.96	5.11
4.99	5.01	5.02	5.01	5.04
Sample 11	**Sample 12**	**Sample 13**	**Sample 14**	**Sample 15**
4.91	4.97	5.09	4.96	4.99
4.93	4.91	4.96	4.99	4.97
5.04	5.02	5.05	4.82	5.01
5.00	4.93	5.12	5.03	4.98
4.90	4.95	5.06	5.00	4.96
4.82	4.96	5.01	4.96	5.02
Sample 16	**Sample 17**	**Sample 18**	**Sample 19**	**Sample 20**
5.01	5.05	4.96	4.90	5.04
5.04	4.97	4.93	4.85	5.03
5.09	5.04	4.97	5.02	4.97
5.07	5.03	5.01	5.01	4.99
5.12	5.09	4.98	4.88	5.05
5.13	5.01	4.92	4.86	5.06

Solution

Compute the value of $\bar{x}$ for each sample and average these values, obtaining $\bar{\bar{x}}$

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \cdots + \bar{x}_{20}}{20}$$

$$= \frac{4.9933 + 4.9583 + 5.0566 + \cdots + 5.0233}{20}$$

$$= \frac{100.043}{20} = 5.002150 \text{ (the centerline)}$$

Compute the values of R and average them, obtaining $\bar{R}$.

$$\bar{R} = \frac{R_1 + R_2 + R_3 + \cdots + R_{20}}{20}$$

$$= \frac{.25 + .12 + .34 + \cdots + .09}{20}$$

$$= \frac{2.72}{20} = .136$$

Determine the value of A_2 by using $n_i = 6$ (size of the sample) from Table A.15: $A_2 = .483$.

The UCL is

$$\bar{\bar{x}} + A_2\bar{R} = 5.00215 + (.483)(.136) = 5.00215 + .06569 = 5.06784$$

The LCL is

$$\bar{\bar{x}} - A_2\bar{R} = 5.00215 - (.483)(.136) = 5.00215 - .06569 = 4.93646$$

Using the standard deviation instead of the range,

$$\bar{s} = \frac{\bar{s}_1 + \bar{s}_2 + \bar{s}_3 + \cdots + \bar{s}_{20}}{20}$$

$$= \frac{.0905 + .0397 + .1136 + \cdots + .0356}{20}$$

$$= .0494$$

Determine the value of A_3 by using $n_i = 6$ (sample size) from Table A.15:

$$A_2 = 1.287$$

The UCL is

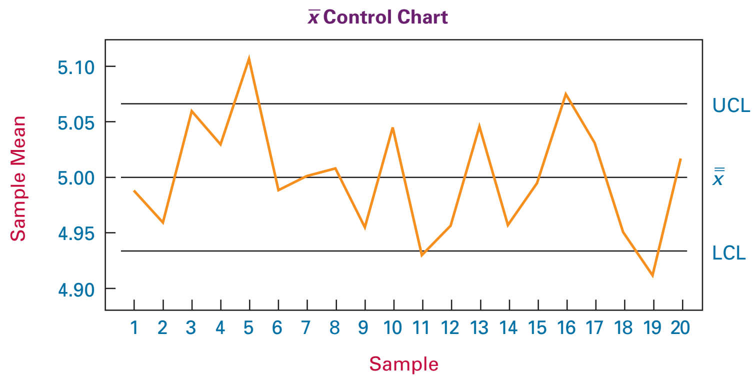
$$\bar{\bar{x}} + A_3\bar{s} = 5.00215 + (1.287)(.0494) = 5.00215 + .06358 = 5.06573$$

The UCL is

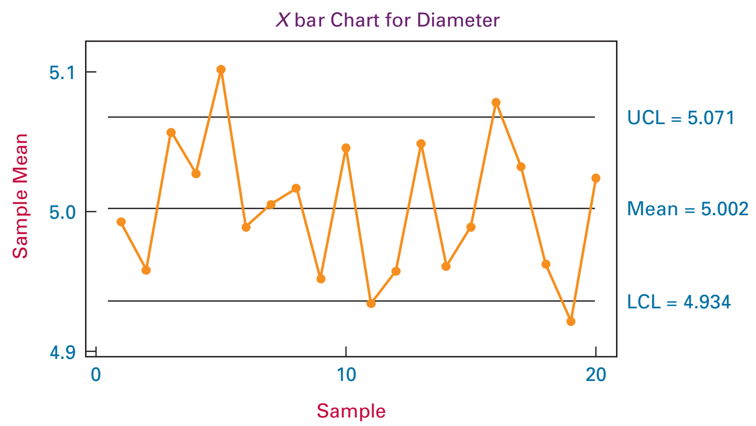
$$\bar{\bar{x}} - A_3\bar{s} = 5.00215 - (1.287)(.0494) = 5.00215 - .06358 = 4.93857$$

The following graph depicts the $\bar{x}$ control chart using the range (rather than the standard deviation) as the measure of dispersion to compute LCL and UCL. Observe that if the standard deviation is used instead of the range to compute LCL and UCL, because of the precision (or lack thereof) of this chart, there is little, if any, perceptible difference in LCL and UCL by the two methods.

Note that the sample means for samples 5 and 16 are above the UCL and the sample means for samples 11 and 19 are below the LCL. This result indicates that these four samples are out of control and alerts the production supervisor or worker to initiate further investigation of bearings produced during these periods. All other samples are within the control limits.



Shown next is the Minitab output for this problem. Note that the Minitab output is nearly identical to the control chart just shown.



R Charts

An ***R* chart** is a plot of the sample ranges and often is used in conjunction with an $\bar{x}$ chart. Whereas $\bar{x}$ charts are used to plot the central tendency values, $\bar{x}$, for each sample, *R* charts are used to plot the variation of each sample as measured by the sample range. The centerline of an *R* chart is the average range, $\bar{R}$. Lower control limits (LCLs) are determined by $D_3\bar{R}$ where D_3 is a weight applied to $\bar{R}$ reflecting sample size. The value of D_3 can be obtained from Table A.15. Upper control limits (UCLs) are determined by $D_4\bar{R}$ where D_4 is a value obtained from Table A.15, which also reflects sample size. The following steps lead to an *R* chart.

1. Decide on the quality to be measured.
2. Determine a sample size.
3. Gather 20 to 30 samples.
4. Compute the sample range, *R*, for each sample.
5. Determine the average sample range for all samples, $\bar{R}$, as

$$\bar{R} = \frac{\Sigma R}{k}$$

   where $k =$ the number of samples.
6. Using the size of the samples, n_i, find the values of D_3 and D_4 in Table A.15.
7. Construct the centerline and control limits.

$$\text{Centerline} = \bar{R}$$
$$\text{UCL} = D_4\bar{R}$$
$$\text{LCL} = D_3\bar{R}$$

DEMONSTRATION PROBLEM 18.2

Construct an *R* chart for the 20 samples of data in Demonstration Problem 18.1 on bearings.

Solution

Compute the sample ranges shown.

Sample	Range
1	.25
2	.12
3	.34
4	.10
5	.07
6	.05
7	.03
8	.09
9	.14
10	.18
11	.22
12	.11
13	.16
14	.21
15	.06
16	.12
17	.12
18	.09
19	.17
20	.09

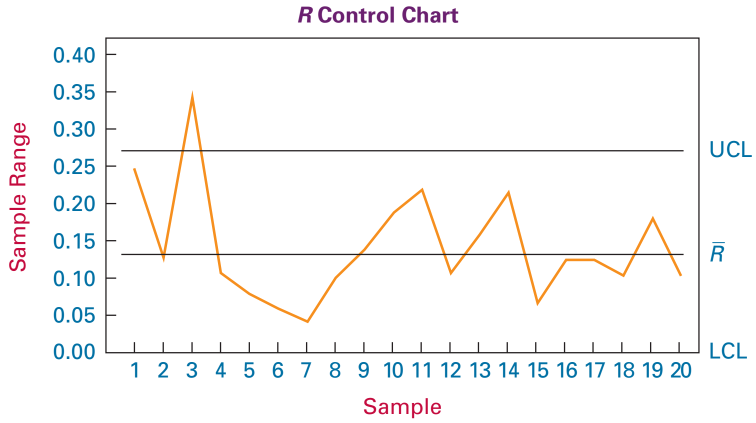
Compute $\bar{R}$

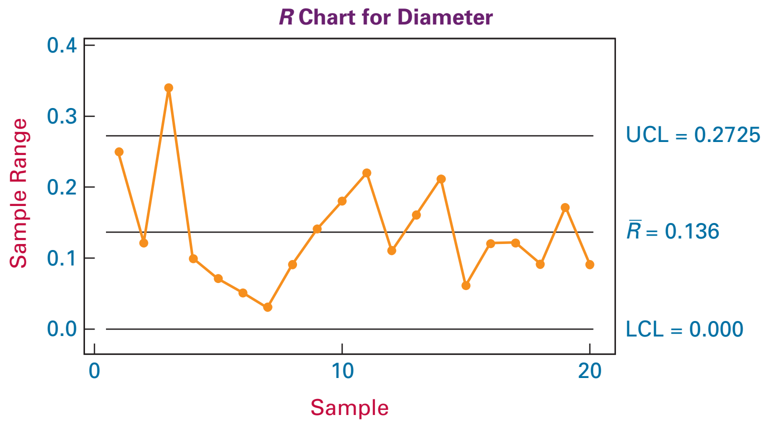
$$\bar{R} = \frac{.25 + .12 + .34 + \cdots + .09}{20} = \frac{2.72}{20} = .136$$

For $n_j = 6$, $D_3 = 0$, and $D_4 = 2.004$ (from Table A.15):

$$\text{Centerline } \bar{R} = .136$$
$$\text{LCL} = D_3\bar{R} = (0)(.136) = 0$$
$$\text{UCL} = D_4\bar{R} = (2.004)(.136) = .2725$$

The resulting R chart for these data is shown next, followed by the Minitab output. Note that the range for sample 3 is out of control (beyond the UCL). The range of values in sample 3 appears to be unacceptable. Further investigation of the population from which this sample was drawn is warranted.





p Charts

When product attributes are measurable, $\bar{x}$ charts and R charts can be formulated from the data. Sometimes, however, product inspection yields no measurement—only a yes-or-no type of conclusion based on whether the item complies with the specifications. For this type of data, no measure is available from which to average or determine the range. However, attribute compliance can be depicted graphically by a p chart. A ***p* chart** *graphs the proportion of sample items in noncompliance for multiple samples.*

For example, suppose a company producing electric motors samples 40 motors three times a week for a month. For each group of 40 motors, it determines the proportion of the sample group that does not comply with the specifications. It then plots these sample proportions, $\hat{p}$, on a p chart to identify trends or samples with unacceptably high proportions of nonconformance. Other p chart applications include determining whether a gallon of paint has been manufactured with acceptable texture, a pane of glass contains cracks, or a tire has a defective tread.

Like the $\bar{x}$ chart and the R chart, a p chart contains a centerline. The centerline is the average of the sample proportions. Upper and lower control limits are computed from the average of the sample proportions plus or minus three standard deviations of proportions. The following are the steps for constructing a p chart.

1. Decide on the quality to be measured.
2. Determine a sample size.
3. Gather 20 to 30 samples.
4. Compute the sample proportion:

$$\hat{p} = \frac{n_{\text{non}}}{n}$$

where

n_{non} = the number of items in the sample in noncompliance
n = the number of items in the sample

5. Compute the average proportion:

$$p = \frac{\Sigma \hat{p}}{k}$$

where

$\hat{p} = \frac{n_{\text{non}}}{n}$ = the sample proportion
k = the number of samples

6. Determine the centerline, UCL, and LCL, when $q = 1 - p$.

$$\text{Centerline} = p$$

$$\text{UCL} = p + 3\sqrt{\frac{p \cdot q}{n}}$$

$$\text{LCL} = p - 3\sqrt{\frac{p \cdot q}{n}}$$

DEMONSTRATION PROBLEM 18.3

A company produces bond paper and, at regular intervals, samples of 50 sheets of paper are inspected. Suppose 20 random samples of 50 sheets of paper each are taken during a certain period of time, with the following numbers of sheets in noncompliance per sample. Construct a p chart from these data.

Sample	n	Out of Compliance
1	50	4
2	50	3
3	50	1
4	50	0
5	50	5
6	50	2
7	50	3
8	50	1
9	50	4
10	50	2
11	50	2
12	50	6
13	50	0
14	50	2
15	50	1
16	50	6
17	50	2
18	50	3
19	50	1
20	50	5

Solution

From the data, $n = 50$. The values of $\hat{p}$ follow.

Sample	$\hat{p}$ (out of compliance)
1	4/50 = .08
2	3/50 = .06
3	1/50 = .02
4	0/50 = .00
5	5/50 = .10
6	2/50 = .04
7	3/50 = .06
8	1/50 = .02
9	4/50 = .08
10	2/50 = .04
11	2/50 = .04
12	6/50 = .12
13	0/50 = .00
14	2/50 = .04
15	1/50 = .02
16	6/50 = .12
17	2/50 = .04
18	3/50 = .06
19	1/50 = .02
20	5/50 = .10

The value of p is obtained by averaging these $\hat{p}$ values.

$$p = \frac{\hat{p}_1 + \hat{p}_2 + \hat{p}_3 + \cdots \hat{p}_{20}}{20}$$

$$= \frac{.08 + .06 + .02 + \cdots + .10}{20} = \frac{1.06}{20} = .053$$

The centerline is $p = .053$.

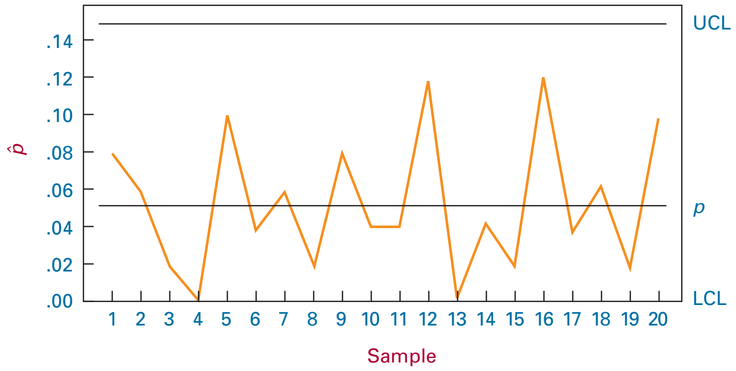
The UCL is

$$p + 3\sqrt{\frac{p \cdot q}{n}} = .053 + 3\sqrt{\frac{(.053)(.947)}{50}} = .053 + .095 = .148$$

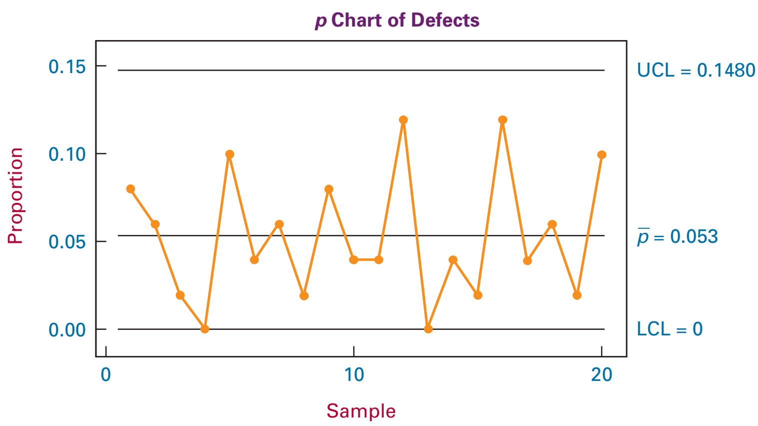
The LCL is

$$p - 3\sqrt{\frac{p \cdot q}{n}} = .053 - 3\sqrt{\frac{(.053)(.947)}{50}} = .053 - .095 = -.042$$

To have −.042 item in noncompliance is impossible, so the lower control limit is 0. Following is the p chart for this problem. Note that all 20 proportions are within the quality-control limits.



Shown next is the Minitab output for this p chart. Note that the computer output is essentially the same as the graph just shown.



c Charts

The c chart is less widely used than the $\bar{x}$, the R, or the p chart. Like the p chart, the c chart attempts to formulate information about defective items. However, whereas the p chart is a control chart that displays the proportion of items in a sample that are out of compliance with specifications, a **c chart** *displays the number of nonconformances per item or unit.* Examples of nonconformances could be paint flaws, scratches, openings drilled too large or too small, or shorts in electric wires. The c chart allows for multiple nonconforming features per item or unit. For example, if an item is a radio, there can be scratches (multiple) in the paint, poor soldering, bad wiring, broken dials, burned-out light bulbs, and broken antennae. A unit need not be an item such as a computer chip. It can be a bolt of cloth, 4 feet of wire, or a 2×4 board. The requirement is that the unit remain consistent throughout the test or experiment.

In computing a c chart, a c value is determined for each item or unit by tallying the total nonconformances for the item or unit. The centerline is computed by averaging the c values for all items or units. Because in theory nonconformances per item or unit are rare, the Poisson distribution is used as the basis for the c chart. The long-run average for the Poisson distribution is λ, and the analogous long-run average for a c chart is $\bar{c}$ (the average of the c values for the items or units studied), which is used as the centerline value. Upper control limits (UCL) and lower control limits (LCL) are computed by adding or subtracting three standard deviations of the mean, $\bar{c}$, from the centerline value, $\bar{c}$. The standard deviation of a Poisson distribution is the square root of λ; likewise, the standard deviation of $\bar{c}$ is the square root of $\bar{c}$. The UCL is thus determined by $\bar{c} + 3\sqrt{\bar{c}}$ and the LCL is given by $\bar{c} - 3\sqrt{\bar{c}}$. The following steps are used for constructing a c chart.

1. Decide on nonconformances to be evaluated.
2. Determine the number of items of units to be studied. (This number should be at least 25.)
3. Gather items or units.
4. Determine the value of c for each item or unit by summing the number of nonconformances in the item or unit.
5. Calculate the value of $\bar{c}$.

$$\bar{c} = \frac{c_1 + c_2 + c_3 + \cdots + c_i}{i}$$

where

i = number of items
c_i = number of nonconformances per item

6. Determine the centerline, UCL, and LCL.

$$\text{Centerline} = \bar{c}$$
$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

DEMONSTRATION PROBLEM 18.4

A manufacturer produces gauges to measure oil pressure. As part of the company's statistical process control, 25 gauges are randomly selected and tested for nonconformances. The results are shown here. Use these data to construct a c chart that displays the nonconformances per item.

Item Number	Number of Nonconformances	Item Number	Number of Nonconformances
1	2	14	2
2	0	15	1
3	3	16	4
4	1	17	0
5	2	18	2
6	5	19	3
7	3	20	2
8	2	21	1
9	0	22	3
10	0	23	2
11	4	24	0
12	3	25	3
13	2		

Solution

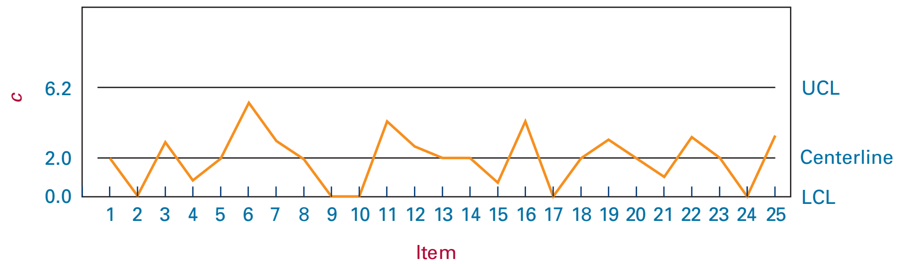
Determine the centerline, UCL, and LCL.

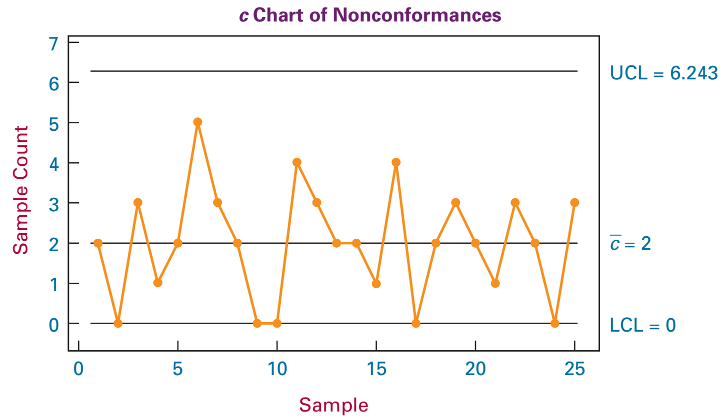
$$\text{Centerline} = \bar{c} = \frac{2 + 0 + 3 + \cdots + 3}{25} = \frac{50}{25} = 2.0$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 2.0 + 3\sqrt{2.0} = 2.0 + 4.2 = 6.2$$

$$\text{LCL} = \bar{c} + 3\sqrt{\bar{c}} = 2.0 - 3\sqrt{2.0} = 2.0 - 4.2 = -2.2$$

The lower control limit cannot be less than zero; thus, the LCL is 0. The graph of the control chart, followed by the Minitab *c* chart, is shown next. Note that none of the points are beyond the control limits and there is a healthy deviation of points both above and below the centerline. This chart indicates a process that is relatively in control, with an average of two nonconformances per item.





Interpreting Control Charts

How can control charts be used to monitor processes? When is a process out of control? An evaluation of the points plotted on a control chart examines several things. Obviously, one concern is points that are outside the control limits. Control chart outer limits (UCL and LCL) are established at three standard deviations above and below the centerline. The empirical rule discussed in Chapter 3 and the *z* table value for $z = 3$ indicate that approximately 99.7% of all values should be within three standard deviations of the mean of the statistic. Applying this rule to control charts suggests that fewer than .3% of all points should be beyond the upper and lower control limits by chance. Thus, one of the more elementary items a control chart observer looks for is points outside LCL and UCL. If the system is "in control," virtually no data points should be outside these limits. Workers responsible for process control should investigate samples in which sample members are outside the LCL and UCL. In the case of the *c* chart, items that are above the UCL line contain an inordinate number of nonconformances in relation to the average. The occurrence of points beyond the control limits call for further investigation.

Several other criteria can be used to determine whether a control chart is plotting a process that is out of control. In general, there *should* be random fluctuation above and below the centerline within the UCL and LCL. However, a process can be out of control if too many consecutive points are above or below the centerline. Eight or more consecutive points on one side of the centerline are considered too many. In addition, if 10 of 11 or 12 of 14 points are on the same side of the center, the process may be out of control.*

Another criterion for process control operators to look for is trends in the control charts. At any point in the process, is a trend emerging in the data? As a rule of thumb, if six or more points are increasing or are decreasing, the process may be out of control.† Such a trend can indicate that points will eventually deviate increasingly from the centerline (the gap between the centerline and the points will increase).

Another concern with control charts is an overabundance of points in the outer one-third of the region between the centerline and the outer limits (LCL and UCL). By a rationalization similar to that imposed on LCL and UCL, the empirical rule and the table of z values show that approximately 95% of all points should be within two standard deviations of the centerline. With this in mind, fewer than 5% of the points should be in the outer one-third of the region between the centerline and the outer control limits (because 95% should be within two-thirds of the region). A rule to follow is that if two out of three consecutive points are in the outer one-third of the chart, a control problem may be present. Likewise, because approximately 68% of all values should be within one standard deviation of the mean (empirical rule, z table for $z = 1$), only 32% should be in the outer two-thirds of the control chart above and below the centerline. As a rule, if four out of five successive points are in the outer two-thirds of the control chart, the process should be investigated.‡

Another consideration in evaluating control charts is the location of the centerline. With each successive batch of samples, it is important to observe whether the centerline is shifting away from specifications.

The following list provides a summary of the control chart abnormalities for which a statistical process controller should be concerned.

1. Points are above UCL and/or below LCL.
2. Eight or more consecutive points are above or below the centerline. Ten out of 11 points are above or below the centerline. Twelve out of 14 points are above or below the centerline.
3. A trend of six or more consecutive points (increasing or decreasing) is present.
4. Two out of three consecutive values are in the outer one-third.
5. Four out of five consecutive values are in the outer two-thirds.
6. The centerline shifts from chart to chart.

Figure 18.7 contains several control charts, each of which has one of these types of problems. The chart in (a) contains points above and below the outer control limits. The one in (b) has eight consecutive points on one side of the centerline. The chart in (c) has seven consecutive increasing points. In (d), at least two out of three consecutive points are in the outer one-third of the control chart. In (e), at least four out of five consecutive points are in the outer two-thirds of the chart.

In investigating control chart abnormalities, several possible causes may be found. Some of them are listed here.§

1. Changes in the physical environment
2. Worker fatigue
3. Worn tools
4. Changes in operators or machines
5. Maintenance

*James R. Evans and William M. Lindsay, *The Management and Control of Quality,* 4th ed. Cincinnati: South-Western College Publishing, 1999.

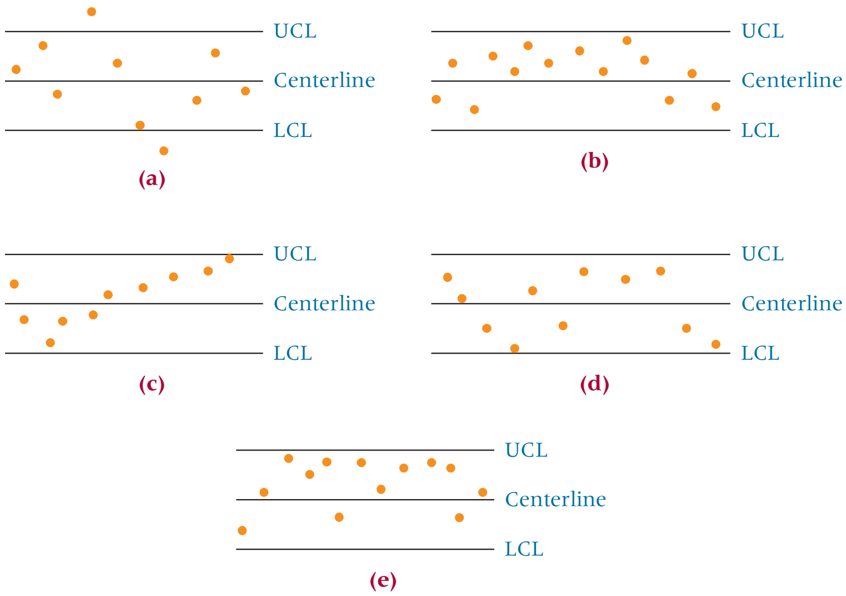
†Richard E. DeVor, Tsong-how Chang, and John W. Sutherland, *Statistical Quality Design and Control*. New York: Macmillan, 1992.

‡DeVor, Chang, and Sutherland; Evans and Lindsay.

§Eugene L. Grant and Richard S. Leavenworth, *Statistical Quality Control*, 5th ed. New York: McGraw-Hill, 1980.

FIGURE 18.7

Control Charts with Problems



6. Changes in worker skills
7. Changes in materials
8. Process modification

The statistical process control person should beware that control chart abnormalities can arise because of measurement errors or incorrect calculation of control limits. Judgment should be exercised so as not to overcontrol the process by readjusting to every oddity that appears to be out of the ordinary on a control chart.

18.3 PROBLEMS

18.4 A food-processing company makes potato chips, pretzels, and cheese chips. Although its products are packaged and sold by weight, the company has been taking sample bags of cheese chips and counting the number of chips in each bag. Shown here is the number of chips per bag for five samples of seven bags of chips. Use these data to construct an $\bar{x}$ chart and an R chart. Discuss the results.

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
25	22	30	32	25
23	21	23	26	23
29	24	22	27	29
31	25	26	28	27
26	23	28	25	27
28	26	27	25	26
27	29	21	31	24

18.5 A toy-manufacturing company has been given a large order for small plastic whistles that will be given away by a large fast-food hamburger chain with its kid's meal. Seven random samples of four whistles have been taken. The weight of each whistle has been ascertained in grams. The data are shown here. Use these data to construct an $\bar{x}$ chart and an R chart. What managerial decisions should be made on the basis of these findings?

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7
4.1	3.6	4.0	4.6	3.9	5.1	4.6
5.2	4.3	4.8	4.8	3.8	4.7	4.4
3.9	3.9	5.1	4.7	4.6	4.8	4.0
5.0	4.6	5.3	4.7	4.9	4.3	4.5

18.6 A machine operator at a pencil-manufacturing facility gathered 10 different random samples of 100 pencils. The operator's inspection was to determine whether the pencils were in compliance or out of compliance with specifications. The results of this inspection are shown below. Use these data to construct a *p* chart. Comment on the results of this chart.

Sample	Size	Number out of Compliance
1	100	2
2	100	7
3	100	4
4	100	3
5	100	3
6	100	5
7	100	2
8	100	0
9	100	1
10	100	6

18.7 A large manufacturer makes valves. Currently it is producing a particular valve for use in industrial engines. As a part of a quality-control effort, the company engineers randomly sample seven groups of 40 valves and inspect them to determine whether they are in or out of compliance. Results are shown here. Use the information to construct a *p* chart. Comment on the chart.

Sample	Size	Number out of Compliance
1	40	1
2	40	0
3	40	1
4	40	3
5	40	2
6	40	5
7	40	2

18.8 A firm in the upper Midwest manufactures light bulbs. Before the bulbs are released for shipment, a sample of bulbs is selected for inspection. Inspectors look for nonconformances such as scratches, weak or broken filaments, incorrectly bored turns, insufficient outside contacts, and others. A sample of thirty five 60-watt bulbs has just been inspected, and the results are shown here. Use these data to construct a *c* chart. Discuss the findings.

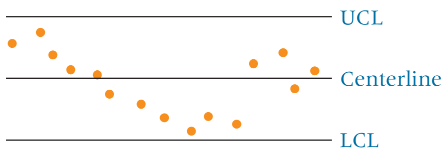
Bulb Number	Number of Nonconformances	Bulb Number	Number of Nonconformances
1	0	19	2
2	1	20	0
3	0	21	0
4	0	22	1
5	3	23	0
6	0	24	0
7	1	25	0
8	0	26	2
9	0	27	0
10	0	28	0
11	2	29	1
12	0	30	0
13	0	31	0
14	2	32	0
15	0	33	0
16	1	34	3
17	3	35	0
18	0		

18.9 A soft drink bottling company just ran a long line of 12-ounce soft drink cans filled with cola. A sample of 32 cans is selected by inspectors looking for nonconforming items. Among the things the inspectors look for are paint defects on the can, improper seal, incorrect volume, leaking contents, incorrect mixture of carbonation and syrup in the soft drink, and out-of-spec syrup mixture. The results of this inspection are given here. Construct a *c* chart from the data and comment on the results.

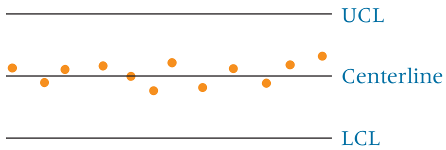
Can Number	Number of Nonconformances	Can Number	Number of Nonconformances
1	2	17	3
2	1	18	1
3	1	19	2
4	0	20	0
5	2	21	0
6	1	22	1
7	2	23	4
8	0	24	0
9	1	25	2
10	3	26	1
11	1	27	1
12	4	28	3
13	2	29	0
14	1	30	1
15	0	31	2
16	1	32	0

18.10 Examine the three control charts shown. Discuss any and all control problems that may be apparent from these control charts.

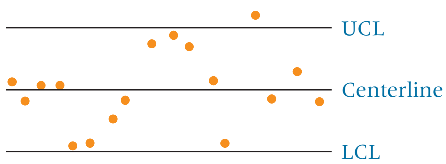
a.



b.

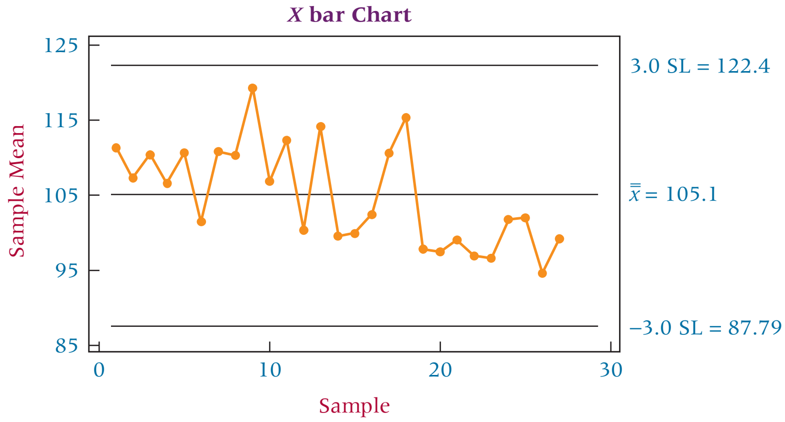


c.



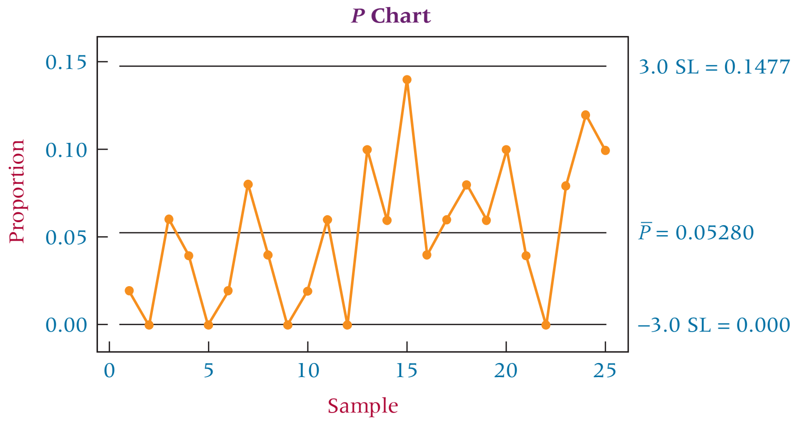
18.11 Study each of the following Minitab control charts and determine whether any of them indicate problems in the processes. Comment on each chart.

a.



b.

c.



DECISION DILEMMA SOLVED

Italy's Piaggio Makes a Comeback

Decision Dilemma Solved

After many years in decline and after several attempts were made to recapture its luster, Piaggio was turned around under the ownership and leadership of Roberto Colaninno. How did he accomplish this? In 2002, one year before Colaninno took over the company, Piaggio had lost 129 million euros and was 577 euros million in debt. Being optimistic and focusing on Piaggio's assets, Colaninno insisted that the company wasn't dying but that "it just needed to be treated better." After making an initial investment of 100 million euros through his holding company for a third of Piaggio and the mandate to run it, he began his run at improving the company. He quickly hired a chief executive, Rocco Sabelli, who led an effort to redesign the factory and its assembly lines. Previously, each of several assembly lines had been set up to produce only one particular scooter model, but because demands for the different models varied according to their popularity, some assembly lines would have significant downtime while others would be running continuously unable to keep up with the demand of strong-selling models. Under Sabelli, each assembly line was redesigned, refitted, and retooled so that any of Piaggio's scooter models could be made on any line with little changeover time. This created increased capacity for the production of hot-selling models to meet demand and at the same time effected a leveling of both manufacturing and human resources in the plant. Given this was a radical departure from the way Piaggio had done business, one might say that Colaninno and Sabelli "reengineered" the company's manufacturing process. An even more dramatic example of reengineering took place following World War II when Piaggio transformed itself from an aeronautical production company to a scooter firm.

It has been a fairly common practice in Italian businesses for management to "keep its distance" from workers. Sabelli in his first days on the job announced to all workers, including assembly-line workers, that they were to let him know personally about any problems or delays in the production process. Giving workers such access to management shortened the lines of communication so as to quickly improve the product and the process. Such an approach may be viewed, at least in spirit, as not unlike the quality circles introduced by the Japanese.

In a move that surprised many, Colaninno did not fire a single worker, thereby gaining support of unions but also reinforcing a notion argued by Deming that poor quality is usually more about such things as tools, training, design, process efficiency, and supplies than it is about the worker. Colaninno based bonuses for both blue-collar workers and management on profit margins and customer satisfaction—a common quality approach in which company employees are empowered to become responsible for product quality as measured by customer satisfaction, units sold, product warrantee and repair claims, increased market share, and others. Colaninno also installed air conditioning in the factory, and productivity began to increase.

Company engineers were given deadlines for design projects in a manner similar to Six Sigma. As a result, the company recently rolled out two world firsts, including a gas–electric hybrid scooter and one with two wheels in front and one in the back for better road grip.

SUMMARY

Quality means many different things to different people. According to one definition, a quality product delivers to the customer those attributes that have been agreed upon by both buyer and seller. Leading quality experts such as Philip B. Crosby, Armand V. Feigenbaum, and David A. Garvin suggest various divergent views on the notion of quality.

Quality control is the collection of strategies, techniques, and actions an organization can use to ensure the production of a quality product. For decades, U.S. companies used after-process quality control, which essentially consisted of inspectors determining whether a product complied with its specifications. During the 1980s, U.S. companies joined Western European and Asian businesses in instituting in-process quality control, which enables the producer to determine weaknesses and flaws during the production process.

Total quality management occurs when all members of an organization—from the CEO to the line worker—are involved in improving quality. One of the main proponents of total quality management was W. Edwards Deming. Deming was known for his cause-and-effect explanation of total quality management in a company, which is sometimes referred to as the Deming chain reaction. In addition, Deming presented 14 points that can lead to improved total quality management. Some other well-known leaders in the quality movement are Joseph Juran, Philip Crosby, Armand Feigenbaum, Kaoru Ishikawa, and Genichi Taguchi.

In addition to total quality management, there are other major quality movements, including Six Sigma, Design for Six Sigma, and lean manufacturing. Six Sigma, besides being a quality movement, is also a methodology and a measurement. A goal of Six Sigma is to have no more than 3.4 defects per million opportunities. It is essentially a philosophy of zero defects achieved through deep root analysis using a variety of statistical tools and methodologies in a team approach. Design for Six Sigma is a quality scheme that places an emphasis on designing a product or process right the first time so that higher sigma levels of quality are possible. Lean manufacturing is a quality-management philosophy that focuses on the reduction of wastes and the elimination of unnecessary steps in an operation or process.

Some important quality concepts include benchmarking, just-in-time inventory systems, reengineering, Failure Mode and Effects Analysis (FMEA), poka-yoke, quality circles, and Six Sigma teams. Benchmarking is a technique through which a company attempts to develop product and process excellence by examining and emulating the best practices and techniques used in the industry. Just-in-time inventory systems are inventory systems that focus on raw materials, subparts, and suppliers. Just-in-time is a philosophy of coordination and cooperation between supplier and manufacturer such that a part or raw material arrives just as it is needed. This approach saves on inventory and also serves as a catalyst for discovering bottlenecks and inefficiencies. It changes the manufacturer-supplier relationship. Reengineering is a radical approach to total quality management in which the core business process is redesigned. Failure Mode and Effects Analysis is a systematic way for identifying the effects of potential product or process failure and includes methodology for eliminating or reducing the chance of failure. Poka-yoke means "mistake proofing," and it uses devices, methods, or inspections to avoid machine error or simple human error. Quality circles are small groups of workers who meet regularly to consider quality issues. Six Sigma teams, lead by a black belt, attempt to uncover root causes of a quality problem or opportunity and through the DMAIC process seek to make significant improvement resulting in substantial savings to the company.

Ishikawa developed seven basic tools of quality that he believed could be used to solve most quality-related problems: flowchart, Pareto chart, cause-and-effect diagram, control chart, check sheet, histogram, and scatter chart. Flowcharts are schematic representations of all activities that occur in a process. Pareto analysis is a method of examining types of defects that occur with a product. The result is usually a vertical bar chart that depicts the most common types of defects ranked in order of occurrence. The cause-and-effect (fishbone) diagram displays potential causes of quality problems. The diagram is shaped like a fish skeleton, with the head being the problem and the skeletal bones being the potential causes. A control chart is a graphic method of evaluating whether a process is or is not in a state of statistical control. Check sheets are simple forms consisting of multiple categories and columns for recording the frequency of outcomes for some quality-related event or activity under study so that data can be presented in a logical format. A histogram is a type of vertical bar chart that is used to depict a frequency distribution. A scatter chart is a graphical mechanism for examining the relationship between two numerical variables.

Control charts are used to monitor product variation, thus enabling operators, technicians, and managers to see when a process gets out of control. The $\bar{x}$ chart and the R chart are two types of control charts for measurements. The $\bar{x}$ chart is a graph of sample means computed on a series of small random samples over time. The R chart is a plot of sample ranges. The $\bar{x}$ chart plots the measure of central tendency, whereas the R chart plots a measure of variability. The p chart and the c chart are two types of control charts for nonconformance. The p chart graphs the proportions of sample items that are in noncompliance. The c chart displays the number of nonconformances per item for a series of sampled items. All four types of control chart are plotted around a centerline and upper and lower control limits. The control limits are located three standard deviations from the centerline.

KEY TERMS

Flash Cards

after-process quality control
benchmarking
c chart
cause-and-effect diagram
centerline
check sheet
control chart
Design for Six Sigma
Failure Mode and Effects Analysis (FMEA)
fishbone diagram
flowchart
histogram
in-process quality control
Ishikawa diagram
just-in-time inventory system
lean manufacturing
lower control limit (LCL)
manufacturing quality
p chart
Pareto analysis
Pareto chart
poka-yoke
process
product quality
quality
quality circle
quality control
R chart
reengineering
scatter chart
Six Sigma
team building
total quality management (TQM)
transcendent quality
upper control limit (UCL)
user quality
value quality
$\bar{x}$ chart

FORMULAS

$\bar{x}$ Charts

$$\text{Centerline: } \bar{\bar{x}} = \frac{\sum \bar{x}}{k}$$

$$\text{UCL: } \bar{\bar{x}} + A_2\bar{R}$$

$$\text{LCL: } \bar{\bar{x}} - A_2\bar{R}$$

or

$$\text{UCL: } \bar{\bar{x}} + A_3\bar{s}$$

$$\text{LCL: } \bar{\bar{x}} - A_3\bar{s}$$

R Charts

$$\text{Centerline: } \bar{R} = \frac{\sum R}{k}$$

$$\text{UCL: } D_4\bar{R}$$

$$\text{LCL: } D_3\bar{R}$$

p Charts

$$\text{Centerline: } p = \frac{\sum \hat{p}}{k}$$

$$\text{UCL: } p + 3\sqrt{\frac{p \cdot q}{n}}$$

$$\text{LCL: } p - 3\sqrt{\frac{p \cdot q}{n}}$$

c Charts

$$\text{Centerline: } \bar{c} = \frac{c_1 + c_2 + c_3 + \cdots + c_i}{i}$$

$$\text{UCL: } \bar{c} + 3\sqrt{\bar{c}}$$

$$\text{LCL: } \bar{c} - 3\sqrt{\bar{c}}$$

ETHICAL CONSIDERATIONS

Unethical or borderline ethical behavior can occur in many areas of total quality management. At the top, CEOs and other high-level managers can profess to the world that the company is committed to quality and not truly promote quality in the organization. Managers who use the quality movement only as a tool for attention and leverage and do not actually intend to implement the process may be acting unethically.

Some of the specifics of quality control and statistical process control lend themselves to unethical behavior. Just-in-time systems can be used as an excuse to implement favoritism among suppliers. With the move to reduce the number of suppliers, contracting managers can be more selective in choosing suppliers. This practice can give contracting agents or purchasing agents more leverage in securing deals through unethical means.

Just-in-time systems often encourage the supplier to do the testing rather than the manufacturer. This self-evaluation opens opportunity for the falsification of records and tests. It is hoped that such behavior is uncovered by just-in-time systems that place pressure on suppliers to ship on-specification parts and materials. The customer or user of the supplies in a just-in-time system is more likely to discover off-specification material than users in traditional systems.

Benchmarking could easily lend itself to violation of patent laws if a company is not careful. It could also encourage business espionage and unfair competitive practices. Benchmarking could create an atmosphere of continually seeking ways to "steal" competitive ideas and innovations.

Control charts present the same potential for unethical behavior as any sampling process. Those workers constructing the charts have opportunity to falsify data, selectively choose favorable items, or graphically misrepresent data to make a system look in control.

The implementation of a sound quality program in a company must be based on teamwork, mutual support, trust, and honesty. Unethical behavior in quality control can set the process back for years, if not permanently. The intent in the quality movement is to bring out the best in people so as to optimize the quality of the product.

SUPPLEMENTARY PROBLEMS

CALCULATING THE STATISTICS

18.12 Create a flowchart from the following sequence of activities: Begin. Flow to activity A. Flow to decision B. If Yes, flow to activity C. If No, flow to activity D. From C flow to activity E and to activity F. From F, flow to decision G. If Yes, flow to decision H. If No at G, stop. At H, if Yes, flow to activity I and on to activity J and then stop. If No at H, flow to activity J and stop. At D, flow to activity K, flow to L, and flow to decision M. If Yes at M, stop. If No at M, flow to activity N, then stop.

18.13 An examination of rejects shows at least 10 problems. A frequency tally of the problems follows. Construct a Pareto chart for these data.

Problem	Frequency
1	673
2	29
3	108
4	379
5	73
6	564
7	12
8	402
9	54
10	202

18.14 A brainstorm session on possible causes of a problem resulted in five possible causes: A, B, C, D, and E. Cause A has three possible subcauses, cause B has four, cause C has two, cause D has five, and cause E has three. Construct a fishbone diagram for this problem and its possible causes.

TESTING YOUR UNDERSTANDING

18.15 A bottled-water company has been randomly inspecting bottles of water to determine whether they are acceptable for delivery and sale. The inspectors are looking at water quality, bottle condition, and seal tightness. A series of 10 random samples of 50 bottles each is taken. Some bottles are rejected. Use the following information on the number of bottles from each batch that were rejected as being out of compliance to construct a p chart.

Sample	N	Number Out of Compliance
1	50	3
2	50	11
3	50	7
4	50	2
5	50	5
6	50	8
7	50	0
8	50	9
9	50	1
10	50	6

18.16 A fruit juice company sells a glass container filled with 24 ounces of cranapple juice. Inspectors are concerned about the consistency of volume of fill in these containers. Every 2 hours for 3 days of production, a sample of five containers is randomly selected and the volume of fill is measured. The results follow.

Sample 1	Sample 2	Sample 3	Sample 4
24.05	24.01	24.03	23.98
24.01	24.02	23.95	24.00
24.02	24.10	24.00	24.01
23.99	24.03	24.01	24.01
24.04	24.08	23.99	24.00

Sample 5	Sample 6	Sample 7	Sample 8
23.97	24.02	24.01	24.08
23.99	24.05	24.00	24.03
24.02	24.01	24.00	24.00
24.01	24.00	23.97	24.05
24.00	24.01	24.02	24.01

Sample 9	Sample 10	Sample 11	Sample 12
24.00	24.00	24.01	24.00
24.02	24.01	23.99	24.05
24.03	24.00	24.02	24.04
24.01	24.00	24.03	24.02
24.01	24.00	24.01	24.00

Use this information to construct $\bar{x}$ and R charts and comment on any samples that are out of compliance.

18.17 A metal-manufacturing company produces sheet metal. Statistical quality-control technicians randomly select sheets to be inspected for blemishes and size problems. The number of nonconformances per sheet is tallied. Shown here are the results of testing 36 sheets of metal. Use the data to construct a c chart. What is the centerline? What is the meaning of the centerline value?

Sheet Number	Number of Noncon-formances	Sheet Number	Number of Noncon-formances
1	4	19	1
2	2	20	3
3	1	21	4
4	1	22	0
5	3	23	2
6	0	24	3
7	4	25	0
8	5	26	0
9	2	27	4
10	1	28	2
11	2	29	5
12	0	30	3
13	5	31	1
14	4	32	2
15	1	33	0
16	2	34	4
17	1	35	2
18	0	36	3

18.18 A manufacturing company produces cylindrical tubes for engines that are specified to be 1.20 centimeters thick. As part of the company's statistical quality-control effort, random samples of four tubes are taken each hour. The tubes are measured to determine whether they are within thickness tolerances. Shown here are the thickness data in centimeters for nine samples of tubes. Use these data to develop an $\bar{x}$ chart and an R chart. Comment on whether or not the process appears to be in control at this point.

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
1.22	1.20	1.21	1.16	1.24
1.19	1.20	1.18	1.17	1.20
1.20	1.22	1.17	1.20	1.21
1.23	1.20	1.20	1.16	1.18

Sample 6	Sample 7	Sample 8	Sample 9
1.19	1.24	1.17	1.22
1.21	1.17	1.23	1.17
1.21	1.18	1.22	1.16
1.20	1.19	1.16	1.19

18.19 A manufacturer produces digital watches. Every 2 hours a sample of six watches is selected randomly to be tested. Each watch is run for exactly 15 minutes and is timed by an accurate, precise timing device. Because of the variation among watches, they do not all run the same. Shown here are the data from eight different samples given in minutes. Use these data to construct $\bar{x}$ and R charts. Observe the results and comment on whether the process is in control.

Sample 1	Sample 2	Sample 3	Sample 4
15.01	15.03	14.96	15.00
14.99	14.96	14.97	15.01
14.99	15.01	14.96	14.97
15.00	15.02	14.99	15.01
14.98	14.97	15.01	14.99
14.99	15.01	14.98	14.96

Sample 5	Sample 6	Sample 7	Sample 8
15.02	15.02	15.03	14.96
15.03	15.01	15.04	14.99
14.99	14.97	15.03	15.02
15.01	15.00	15.00	15.01
15.02	15.01	15.01	14.98
15.01	14.99	14.99	15.02

18.20 A company produces outdoor home thermometers. For a variety of reasons, a thermometer can be tested and found to be out of compliance with company specification. The company takes samples of thermometers on a regular basis and tests each one to determine whether it meets company standards. Shown here are data from 12 different random samples of 75 thermometers. Use these data to construct a *p* chart. Comment on the pattern of points in the chart.

Sample	*n*	Number Out of Compliance
1	75	9
2	75	3
3	75	0
4	75	2
5	75	7
6	75	14
7	75	11
8	75	8
9	75	5
10	75	4
11	75	0
12	75	7

18.21 A plastics company makes thousands of plastic bottles for another company that manufactures saline solution for users of soft contact lenses. The plastics company randomly inspects a sample of its bottles as part of its quality-control program. Inspectors look for blemishes on the bottle, size and thickness, ability to close, leaks, labeling problems, and so on. Shown here are the results of tests completed on 25 bottles. Use these data to construct a *c* chart. Observe the results and comment on the chart.

Bottle Number	Number of Noncon-formances	Bottle Number	Number of Noncon-formances
1	1	14	0
2	0	15	0
3	1	16	0
4	0	17	1
5	0	18	0
6	2	19	0
7	1	20	1
8	1	21	0
9	0	22	1
10	1	23	2
11	0	24	0
12	2	25	1
13	1		

18.22 A bathtub manufacturer closely inspects several tubs on every shift for nonconformances such as leaks, lack of symmetry, unstable base, drain malfunctions, and so on. The following list gives the number of nonconformances per tub for 40 tubs. Use these data to construct a *c* chart of nonconformances for bathtubs. Comment on the results of this chart.

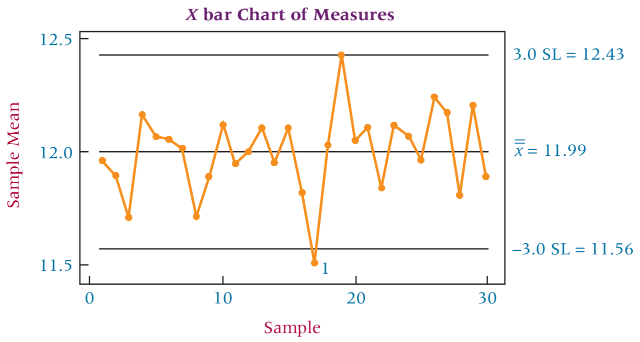
Tub	Number of Noncon-formances	Tub	Number of Noncon-formances
1	3	21	2
2	2	22	5
3	3	23	1
4	1	24	3
5	4	25	4
6	2	26	3
7	2	27	2
8	1	28	0
9	4	29	1
10	2	30	0
11	3	31	2
12	0	32	1
13	3	33	2
14	2	34	1
15	2	35	1
16	1	36	1
17	0	37	3
18	4	38	0
19	3	39	1
20	2	40	4

18.23 A glass manufacturer produces hand mirrors. Each mirror is supposed to meet company standards for such things as glass thickness, ability to reflect, size of handle, quality of glass, color of handle, and so on. To control for these features, the company quality people randomly sample 40 mirrors every shift and determine how many of the mirrors are out of compliance on at least one feature. Shown here are the data for 15 such samples. Use the data to construct a *p* chart. Observe the results and comment on the control of the process as indicated by the chart.

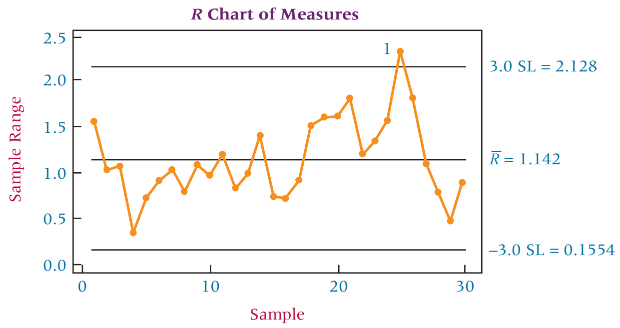
Sample	*n*	Number Out of Compliance
1	40	2
2	40	0
3	40	6
4	40	3
5	40	1
6	40	1
7	40	5
8	40	0
9	40	4
10	40	3
11	40	2
12	40	2
13	40	6
14	40	1
15	40	0

INTERPRETING THE OUTPUT

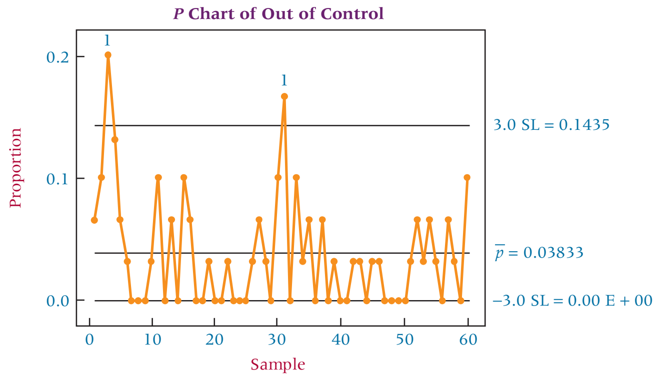
18.24 Study the Minitab chart on the fill of a product that is supposed to contain 12 ounces. Does the process appear to be out of control? Why or why not?



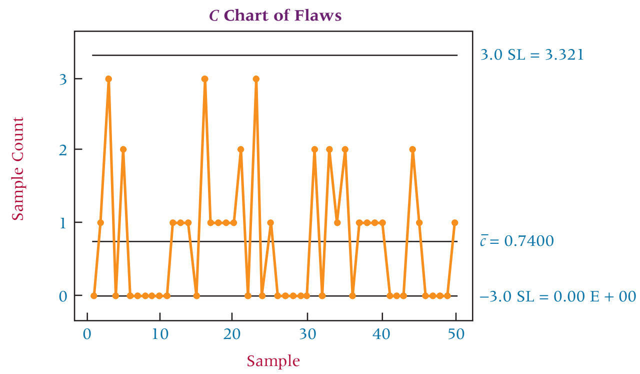
18.25 Study the Minitab *R* chart for the product and data used in Problem 18.24. Comment on the state of the production process for this item.



18.26 Study the Minitab *p* chart for a manufactured item. The chart represents the results of testing 30 items at a time for compliance. Sixty different samples were taken for this chart. Discuss the results and the implications for the production process.



18.27 Study the Minitab *c* chart for nonconformances for a part produced in a manufacturing process. Comment on the results.



ANALYZING THE DATABASES

see www.wiley.com/college/black and WileyPLUS

1. A dairy company in the Manufacturing database tests its quart milk container fills for volume in four-container samples. Shown here are the results of 10 such samples and the volume measurements in quarts. Use the information to construct both an $\bar{x}$ and an *R* chart for the data. Discuss the results. What are the centerline, LCL, and UCL for each of these charts?

Sample Number	Measurements			
1	.98	1.01	1.05	1.03
2	1.02	.94	.97	1.02
3	1.11	1.02	.93	1.01
4	.95	.98	1.02	.96
5	1.03	1.01	1.01	.95
6	1.04	.93	.91	.96
7	.94	1.12	1.10	1.03
8	1.03	.92	.98	1.03
9	1.01	1.01	.99	1.00
10	1.05	.96	1.00	1.04

2. A hospital in the Hospital database takes weekly samples of patient account statements for 12 weeks with each sample containing 40 accounts. Auditors analyze the account statements, looking for nonconforming statements. Shown here are the results of the 12 samples. Use these data to construct a *p* chart for proportion of nonconforming statements. What is the centerline? What are UCL and LCL? Comment on the control chart.

Sample	Number of Nonconforming Statements
1	1
2	0
3	6
4	3
5	0
6	2
7	8
8	3
9	5
10	2
11	2
12	1

CASE

ROBOTRON-ELOTHERM

The Robotron-Elotherm company is known for its expertise in power electronics for welding, induction bonding, and process heating. Originally known as Robotron, the company manufactured bonding products for the automotive industry for more than two decades in the 1960s and 1970s. For several years, Robotron felt it produced and delivered a high-quality product because it rarely received complaints from customers. However, early in the 1980s General Motors gave Robotron an order for induction bonding machines to cure adhesive in auto door joints. Actually, the machines were shipped to a Michigan plant of Sanyo Manufacturing that GM was using as a door builder.

The Japanese firm was unhappy with the quality of the machines. Robotron president Leonard Brzozowski went to the Sanyo plant to investigate the situation in person and learned that the Japanese had a much higher quality standard than the usual American customers. Tolerances were much smaller and inspection was more demanding. Brzozowski said that he realized for the first time that the philosophy, engineering, management, and shop practices of Robotron did not qualify the company for world competition. Brzozowski said that this was the most embarrassing time of his professional career. What should Robotron do about this situation?

Brzozowski began by sending a group of hourly employees to the Sanyo plant. There they met the customer and heard firsthand the many complaints about their product. The workers could see the difference in quality between their machines and those of Sanyo. The plant visit was extremely effective. On the way home, the Robotron workers started discussing what they could do to improve quality.

The company took several steps to begin the process of quality improvement. It established new inspection procedures, bought more accurate inspection tools, changed internal control procedures, and developed baselines against which to measure progress. Teams were organized and sent out to customers 6 months after a purchase to determine customer satisfaction. A hotline was established for customers to call to report product dissatisfaction.

For 1 month, engineers assembled machinery in the shop under the direction of hourly employees. This exercise gave the engineers a new awareness of the importance of accurate, clear drawings; designing smaller, lighter weight details; and minimizing the number of machined surfaces.

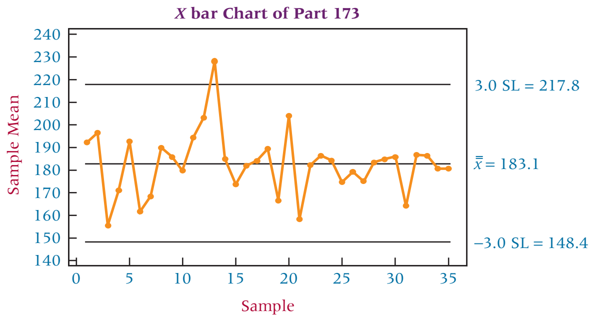
Robotron's effort paid off handsomely. Claims under warranty dropped 40% in 3 years, during which time orders rose at a compound annual rate of 13.5%. The company cut costs and streamlined procedures and processes. Sales increased and new markets opened.

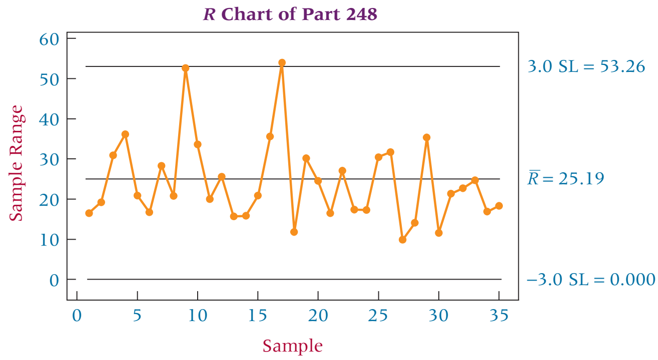
In 1997, Robotron received ISO-9001 certification. Early in 1998, Robotron merged with ELOTHERM, a European company, so that Robotron could more easily enjoy a presence in the European market and at the same time provide ELOTHERM opportunities in North America. Robotron's bonding business expanded into induction heating systems, heat treating, tube welding, and electrical discharge machines. The company maintains a quality system that takes corrective action in response to customer complaints, employee suggestions, or supplier defects. Robotron-ELOTHERM, now SMS ELOTHERM, is part of the SMS worldwide group; and its North American division is located in Cranberry Township, Pennsylvania.

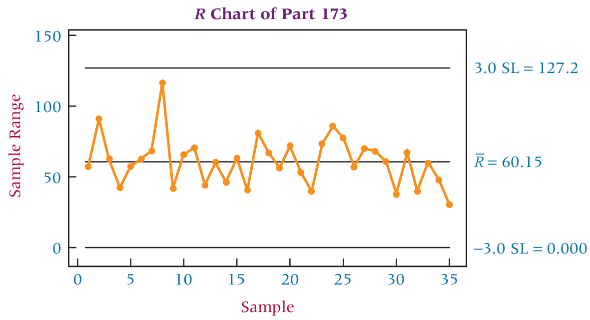
Discussion

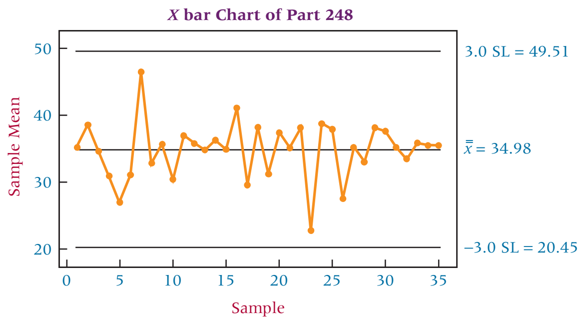
1. As a part of quality improvement, it is highly likely that Robotron analyzed its manufacturing processes. Suppose that as Robotron improved quality, the company wanted to examine other processes including the flow of work orders from the time they are received until they are filled. Use the following verbal sketch of some of the activities that might take place in such a flow as a start, and add some of your own ideas as you draw a flowchart for the process of work orders.

   Work order flow: Received at mailroom. Sent to order processing office. Examined by order processing clerk who decides whether the item is a standard item or a custom item. If it is a standard item, the order is sent to the warehouse. If the item is available, the item is shipped and the order is sent to the billing department. If the item is not available, the order is sent to the plant where it is received by a manufacturing clerk. The clerk checks to determine whether such an item is being manufactured. If so, the order is sent to the end of the assembly line where it will be tagged with one such item. If not, the order is sent to the beginning of the assembly line and flows along the assembly line with the item as it is being made. In either case, when the part comes off the assembly line, the order is attached to the item and sent to shipping. The shipping clerk then ships the item and sends the order to billing. If the ordered item is a customized part, the order is sent straight from the order processing clerk to manufacturing where it follows the same procedures as already described for standard items that have not been manufactured yet.
2. Virtually all quality manufacturers use some type of control chart to monitor performance. Suppose the Minitab control charts shown here are for two different parts produced by Robotron during a particular period. Part 173 is specified to weigh 183 grams. Part 248 contains an opening that is specified to be 35 millimeters in diameter. Study these charts and report to Robotron what you found. Is there any reason for concern? Is everything in control?

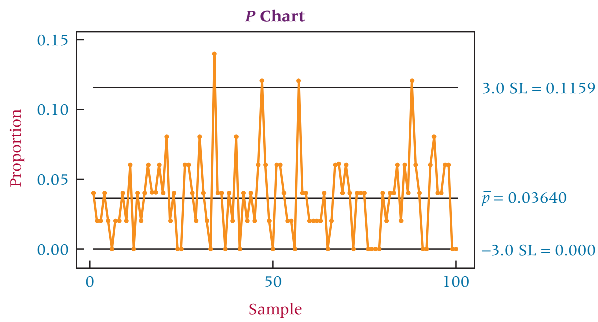








3. Suppose Robotron also keeps p charts on nonconformance. The Minitab chart shown here represents the proportion of nonconforming items for a given part over 100 samples. Study the chart and write a brief report to Robotron about what you learned from the chart. Think about overall performance, out-of-control samples, samples that have outstanding performance, and any general trends that you see.



Source: Adapted from "ROBOTRON: Qualifying for Global Competition," Real-World Lessons for America's Small Businesses: Insights from the Blue Chip Enterprise Initiative. Published by *Nation's Business* magazine on behalf of Connecticut Mutual Life Insurance Company and the U.S. Chamber of Commerce in association with the Blue Chip Enterprise Initiative 1994. See also Robotron, available at http://www.robotron.com/aboutus/iso9001.html; http://www.robotron.com/products; http://www.robotron.com/aboutus/merger.html; http://www.robotron.com/aboutus/jobs.html.

USING THE COMPUTER

MINITAB

Minitab has the capability of producing several different types of quality charts and graphs. All of the quality analyses begin with the selection of **Stat** from the menu bar. From the pulldown menu that appears, two options can be used to generate quality analyses, **Control Charts** and **Quality Tools**.

- The **Control Charts** pulldown menu offers several options. Here we focus on the four types of control charts discussed in the chapter. The $\bar{x}$ and the R charts are accessed by selecting **Variables Charts for Subgroups**. The p and the c charts are accessed by selecting **Attributes Charts**.
- To construct an $\bar{x}$ chart, select **Xbar** from the **Variables Charts for Subgroups** pulldown menu bar. In the first space, select **All observations for a chart are in one column** if the data are in one or more columns and enter the columns in the next space. Select **Observations for a subgroup are in one row of columns** if subgroups are arranged in rows across several columns and enter the columns in the next space. When using **All observations for a chart are in one column**, in **Subgroup sizes**, enter a number or a column of subscripts. Select **Scale** to input Time, Axes and Ticks, Gridlines, and Reference Lines. Select **Labels** to input titles, and subtitles. Select **MultipleGraphs** to set the same

y-scale for different variables. Select **Data Options** to specify which rows to include or exclude. Select **Xbar Options** to specify a mean and/or a standard deviation rather than have Minitab compute them for you. In addition, there are options for parameters, methods for estimating the standard deviation, computing control limits (S Limits), setting limits for determining when the process is out of control, defining stages, using a Box-Cox transformation, displaying output, and storage.

- To construct an *R* chart, select **Rbar** from the **Variables Charts for Subgroups** pulldown menu bar. In the first space, select **All observations for a chart are in one column** if the data are in one or more columns and enter the columns in the next space. Select **Observations for a subgroup are in one row of columns** if subgroups are arranged in rows across several columns and enter the columns in the next space. When using **All observations for a chart are in one column**, in **Subgroup sizes**, enter a number or a column of subscripts. Select **Scale** to input Time, Axes and Ticks, Gridlines, and Reference Lines. Select **Labels** to input titles, and subtitles. Select **Multiple Graphs** to set the same *y*-scale for different variables. Select **Data Options** to specify which rows to include or exclude. Select **Rbar Options** to specify a standard deviation rather than have Minitab compute it for you. In addition, there are options for parameters, methods for estimating the standard deviation, computing control limits (S Limits), setting limits for determining when the process is out of control, defining stages, using a Box-Cox transformation, displaying output, and storage.
- To construct a *p* chart, select **P** from the **Attributes Charts** pulldown menu bar. In the **Variables** space, enter the columns that contain the number of defectives for each sample. In the **Subgroup sizes** space, enter a number or a column containing the sizes. Select **Scale** to input Time, Axes and Ticks, Gridlines, and Reference Lines. Select **Labels** to input titles, and subtitles. Select **Multiple Graphs** to set the same *y*-scale for different variables. Select **Data Options** to specify which rows to include or exclude. Select **P Chart Options** to specify a value for the proportion rather than have Minitab estimate it from the data. In addition, there are options estimating for parameters, computing control limits (S Limits), setting limits for determining when the process is out of control, defining stages, displaying output, and storage.
- To construct a *c* chart, select **C** from the **Attributes Charts** pulldown menu bar. In the **Variables** space, enter the columns that contain the number of defectives for each sample. Select **Scale** to input Time, Axes and Ticks, Gridlines, and Reference Lines. Select **Labels** to input titles, and subtitles. Select **Multiple Graphs** to set the same *y*-scale for different variables. Select **Data Options** to specify which rows to include or exclude. Select **C Chart Options** to specify a value for the mean rather than have Minitab estimate it from the data. In addition, there are options estimating for parameters, computing control limits (S Limits), setting limits for determining when the process is out of control, defining stages, displaying output, and storage.

The **Quality Tools** pulldown menu offers several options. Here we focus on two: the Pareto chart and the cause-and-effect (fishbone) diagram.

- To construct a Pareto Chart, select **Pareto Chart**. If the chart defects are individually entered rather than tallied, check **Chart defects data in**. If the labels (problems or types of defects) are in one column and the frequencies are in another column, check **Chart defects table**. Place the column containing the labels in **Labels in**. Place the column containing the frequencies in **Frequencies in**. By checking **combine defects after the first__% into one**, you have the option of collapsing all of the rest of the smaller categories into one after a certain percentage of defect problems have been identified. With **Options**, you can specify a title and *x*- and *y*-axis labels.
- To construct a cause-and-effect diagram, select **Cause-and-Effect**. Note: You can change or add branches or sub-branches after you create the diagram. If you leave the **Causes** spaces empty, Minitab will display the diagram without any sub-branches. If you want to enter subcauses, enter the column containing a list of the subcauses in the space under **Causes** on line with the associated label. For example, if you have four subcauses for **Machines** in C1, enter C1 in the space beside **Machines** and Minitab will create sub-branches associated with **Machines**. Under **Label**, type the label that you want to display to change the default branch labels right over the default label. To display a blank diagram, check **Do not label the branches**. To suppress empty branches, check **Do not display empty branches**.

APPENDIX A

Tables

TABLE A.1

Random Numbers

12651	61646	11769	75109	86996	97669	25757	32535	07122	76763
81769	74436	02630	72310	45049	18029	07469	42341	98173	79260
36737	98863	77240	76251	00654	64688	09343	70278	67331	98729
82861	54371	76610	94934	72748	44124	05610	53750	95938	01485
21325	15732	24127	37431	09723	63529	73977	95218	96074	42138
74146	47887	62463	23045	41490	07954	22597	60012	98866	90959
90759	64410	54179	66075	61051	75385	51378	08360	95946	95547
55683	98078	02238	91540	21219	17720	87817	41705	95785	12563
79686	17969	76061	83748	55920	83612	41540	86492	06447	60568
70333	00201	86201	69716	78185	62154	77930	67663	29529	75116
14042	53536	07779	04157	41172	36473	42123	43929	50533	33437
59911	08256	06596	48416	69770	68797	56080	14223	59199	30162
62368	62623	62742	14891	39247	52242	98832	69533	91174	57979
57529	97751	54976	48957	74599	08759	78494	52785	68526	64618
15469	90574	78033	66885	13936	42117	71831	22961	94225	31816
18625	23674	53850	32827	81647	80820	00420	63555	74489	80141
74626	68394	88562	70745	23701	45630	65891	58220	35442	60414
11119	16519	27384	90199	79210	76965	99546	30323	31664	22845
41101	17336	48951	53674	17880	45260	08575	49321	36191	17095
32123	91576	84221	78902	82010	30847	62329	63898	23268	74283
26091	68409	69704	82267	14751	13151	93115	01437	56945	89661
67680	79790	48462	59278	44185	29616	76531	19589	83139	28454
15184	19260	14073	07026	25264	08388	27182	22557	61501	67481
58010	45039	57181	10238	36874	28546	37444	80824	63981	39942
56425	53996	86245	32623	78858	08143	60377	42925	42815	11159
82630	84066	13592	60642	17904	99718	63432	88642	37858	25431
14927	40909	23900	48761	44860	92467	31742	87142	03607	32059
23740	22505	07489	85986	74420	21744	97711	36648	35620	97949
32990	97446	03711	63824	07953	85965	87089	11687	92414	67257
05310	24058	91946	78437	34365	82469	12430	84754	19354	72745
21839	39937	27534	88913	49055	19218	47712	67677	51889	70926
08833	42549	93981	94051	28382	83725	72643	64233	97252	17133
58336	11139	47479	00931	91560	95372	97642	33856	54825	55680
62032	91144	75478	47431	52726	30289	42411	91886	51818	78292
45171	30557	53116	04118	58301	24375	65609	85810	18620	49198
91611	62656	60128	35609	63698	78356	50682	22505	01692	36291
55472	63819	86314	49174	93582	73604	78614	78849	23096	72825
18573	09729	74091	53994	10970	86557	65661	41854	26037	53296
60866	02955	90288	82136	83644	94455	06560	78029	98768	71296
45043	55608	82767	60890	74646	79485	13619	98868	40857	19415
17831	09737	79473	75945	28394	79334	70577	38048	03607	06932
40137	03981	07585	18128	11178	32601	27994	05641	22600	86064
77776	31343	14576	97706	16039	47517	43300	59080	80392	63189
69605	44104	40103	95635	05635	81673	68657	09559	23510	95875
19916	52934	26499	09821	97331	80993	61299	36979	73599	35055
02606	58552	07678	56619	65325	30705	99582	53390	46357	13244
65183	73160	87131	35530	47946	09854	18080	02321	05809	04893
10740	98914	44916	11322	89717	88189	30143	52687	19420	60061
98642	89822	71691	51573	83666	61642	46683	33761	47542	23551
60139	25601	93663	25547	02654	94829	48672	28736	84994	13071

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TABLE A.2

Binomial Probability Distribution

n = 1

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.900	.800	.700	.600	.500	.400	.300	.200	.100
1	.100	.200	.300	.400	.500	.600	.700	.800	.900

n = 2

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.810	.640	.490	.360	.250	.160	.090	.040	.010
1	.180	.320	.420	.480	.500	.480	.420	.320	.180
2	.010	.040	.090	.160	.250	.360	.490	.640	.810

n = 3

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.729	.512	.343	.216	.125	.064	.027	.008	.001
1	.243	.384	.441	.432	.375	.288	.189	.096	.027
2	.027	.096	.189	.288	.375	.432	.441	.384	.243
3	.001	.008	.027	.064	.125	.216	.343	.512	.729

n = 4

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.656	.410	.240	.130	.063	.026	.008	.002	.000
1	.292	.410	.412	.346	.250	.154	.076	.026	.004
2	.049	.154	.265	.346	.375	.346	.265	.154	.049
3	.004	.026	.076	.154	.250	.346	.412	.410	.292
4	.000	.002	.008	.026	.063	.130	.240	.410	.656

n = 5

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.590	.328	.168	.078	.031	.010	.002	.000	.000
1	.328	.410	.360	.259	.156	.077	.028	.006	.000
2	.073	.205	.309	.346	.313	.230	.132	.051	.008
3	.008	.051	.132	.230	.313	.346	.309	.205	.073
4	.000	.006	.028	.077	.156	.259	.360	.410	.328
5	.000	.000	.002	.010	.031	.078	.168	.328	.590

n = 6

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.531	.262	.118	.047	.016	.004	.001	.000	.000
1	.354	.393	.303	.187	.094	.037	.010	.002	.000
2	.098	.246	.324	.311	.234	.138	.060	.015	.001
3	.015	.082	.185	.276	.313	.276	.185	.082	.015
4	.001	.015	.060	.138	.234	.311	.324	.246	.098
5	.000	.002	.010	.037	.094	.187	.303	.393	.354
6	.000	.000	.001	.004	.016	.047	.118	.262	.531

(*Continued*)

TABLE A.2

Binomial Probability Distribution (*Continued*)

$n = 7$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.478	.210	.082	.028	.008	.002	.000	.000	.000
1	.372	.367	.247	.131	.055	.017	.004	.000	.000
2	.124	.275	.318	.261	.164	.077	.025	.004	.000
3	.023	.115	.227	.290	.273	.194	.097	.029	.003
4	.003	.029	.097	.194	.273	.290	.227	.115	.023
5	.000	.004	.025	.077	.164	.261	.318	.275	.124
6	.000	.000	.004	.017	.055	.131	.247	.367	.372
7	.000	.000	.000	.002	.008	.028	.082	.210	.478

$n = 8$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.430	.168	.058	.017	.004	.001	.000	.000	.000
1	.383	.336	.198	.090	.031	.008	.001	.000	.000
2	.149	.294	.296	.209	.109	.041	.010	.001	.000
3	.033	.147	.254	.279	.219	.124	.047	.009	.000
4	.005	.046	.136	.232	.273	.232	.136	.046	.005
5	.000	.009	.047	.124	.219	.279	.254	.147	.033
6	.000	.001	.010	.041	.109	.209	.296	.294	.149
7	.000	.000	.001	.008	.031	.090	.198	.336	.383
8	.000	.000	.000	.001	.004	.017	.058	.168	.430

$n = 9$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.387	.134	.040	.010	.002	.000	.000	.000	.000
1	.387	.302	.156	.060	.018	.004	.000	.000	.000
2	.172	.302	.267	.161	.070	.021	.004	.000	.000
3	.045	.176	.267	.251	.164	.074	.021	.003	.000
4	.007	.066	.172	.251	.246	.167	.074	.017	.001
5	.001	.017	.074	.167	.246	.251	.172	.066	.007
6	.000	.003	.021	.074	.164	.251	.267	.176	.045
7	.000	.000	.004	.021	.070	.161	.267	.302	.172
8	.000	.000	.000	.004	.018	.060	.156	.302	.387
9	.000	.000	.000	.000	.002	.010	.040	.134	.387

$n = 10$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.349	.107	.028	.006	.001	.000	.000	.000	.000
1	.387	.268	.121	.040	.010	.002	.000	.000	.000
2	.194	.302	.233	.121	.044	.011	.001	.000	.000
3	.057	.201	.267	.215	.117	.042	.009	.001	.000
4	.011	.088	.200	.251	.205	.111	.037	.006	.000
5	.001	.026	.103	.201	.246	.201	.103	.026	.001
6	.000	.006	.037	.111	.205	.251	.200	.088	.011
7	.000	.001	.009	.042	.117	.215	.267	.201	.057
8	.000	.000	.001	.011	.044	.121	.233	.302	.194
9	.000	.000	.000	.002	.010	.040	.121	.268	.387
10	.000	.000	.000	.000	.001	.006	.028	.107	.349

TABLE A.2

Binomial Probability Distribution (*Continued*)

$n = 11$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.314	.086	.020	.004	.000	.000	.000	.000	.000
1	.384	.236	.093	.027	.005	.001	.000	.000	.000
2	.213	.295	.200	.089	.027	.005	.001	.000	.000
3	.071	.221	.257	.177	.081	.023	.004	.000	.000
4	.016	.111	.220	.236	.161	.070	.017	.002	.000
5	.002	.039	.132	.221	.226	.147	.057	.010	.000
6	.000	.010	.057	.147	.226	.221	.132	.039	.002
7	.000	.002	.017	.070	.161	.236	.220	.111	.016
8	.000	.000	.004	.023	.081	.177	.257	.221	.071
9	.000	.000	.001	.005	.027	.089	.200	.295	.213
10	.000	.000	.000	.001	.005	.027	.093	.236	.384
11	.000	.000	.000	.000	.000	.004	.020	.086	.314

$n = 12$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.282	.069	.014	.002	.000	.000	.000	.000	.000
1	.377	.206	.071	.017	.003	.000	.000	.000	.000
2	.230	.283	.168	.064	.016	.002	.000	.000	.000
3	.085	.236	.240	.142	.054	.012	.001	.000	.000
4	.021	.133	.231	.213	.121	.042	.008	.001	.000
5	.004	.053	.158	.227	.193	.101	.029	.003	.000
6	.000	.016	.079	.177	.226	.177	.079	.016	.000
7	.000	.003	.029	.101	.193	.227	.158	.053	.004
8	.000	.001	.008	.042	.121	.213	.231	.133	.021
9	.000	.000	.001	.012	.054	.142	.240	.236	.085
10	.000	.000	.000	.002	.016	.064	.168	.283	.230
11	.000	.000	.000	.000	.003	.017	.071	.206	.377
12	.000	.000	.000	.000	.000	.002	.014	.069	.282

$n = 13$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.254	.055	.010	.001	.000	.000	.000	.000	.000
1	.367	.179	.054	.011	.002	.000	.000	.000	.000
2	.245	.268	.139	.045	.010	.001	.000	.000	.000
3	.100	.246	.218	.111	.035	.006	.001	.000	.000
4	.028	.154	.234	.184	.087	.024	.003	.000	.000
5	.006	.069	.180	.221	.157	.066	.014	.001	.000
6	.001	.023	.103	.197	.209	.131	.044	.006	.000
7	.000	.006	.044	.131	.209	.197	.103	.023	.001
8	.000	.001	.014	.066	.157	.221	.180	.069	.006
9	.000	.000	.003	.024	.087	.184	.234	.154	.028
10	.000	.000	.001	.006	.035	.111	.218	.246	.100
11	.000	.000	.000	.001	.010	.045	.139	.268	.245
12	.000	.000	.000	.000	.002	.011	.054	.179	.367
13	.000	.000	.000	.000	.000	.001	.010	.055	.254

(*Continued*)

TABLE A.2

Binomial Probability Distribution (*Continued*)

$n = 14$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.229	.044	.007	.001	.000	.000	.000	.000	.000
1	.356	.154	.041	.007	.001	.000	.000	.000	.000
2	.257	.250	.113	.032	.006	.001	.000	.000	.000
3	.114	.250	.194	.085	.022	.003	.000	.000	.000
4	.035	.172	.229	.155	.061	.014	.001	.000	.000
5	.008	.086	.196	.207	.122	.041	.007	.000	.000
6	.001	.032	.126	.207	.183	.092	.023	.002	.000
7	.000	.009	.062	.157	.209	.157	.062	.009	.000
8	.000	.002	.023	.092	.183	.207	.126	.032	.001
9	.000	.000	.007	.041	.122	.207	.196	.086	.008
10	.000	.000	.001	.014	.061	.155	.229	.172	.035
11	.000	.000	.000	.003	.022	.085	.194	.250	.114
12	.000	.000	.000	.001	.006	.032	.113	.250	.257
13	.000	.000	.000	.000	.001	.007	.041	.154	.356
14	.000	.000	.000	.000	.000	.001	.007	.044	.229

$n = 15$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.206	.035	.005	.000	.000	.000	.000	.000	.000
1	.343	.132	.031	.005	.000	.000	.000	.000	.000
2	.267	.231	.092	.022	.003	.000	.000	.000	.000
3	.129	.250	.170	.063	.014	.002	.000	.000	.000
4	.043	.188	.219	.127	.042	.007	.001	.000	.000
5	.010	.103	.206	.186	.092	.024	.003	.000	.000
6	.002	.043	.147	.207	.153	.061	.012	.001	.000
7	.000	.014	.081	.177	.196	.118	.035	.003	.000
8	.000	.003	.035	.118	.196	.177	.081	.014	.000
9	.000	.001	.012	.061	.153	.207	.147	.043	.002
10	.000	.000	.003	.024	.092	.186	.206	.103	.010
11	.000	.000	.001	.007	.042	.127	.219	.188	.043
12	.000	.000	.000	.002	.014	.063	.170	.250	.129
13	.000	.000	.000	.000	.003	.022	.092	.231	.267
14	.000	.000	.000	.000	.000	.005	.031	.132	.343
15	.000	.000	.000	.000	.000	.000	.005	.035	.206

TABLE A.2

Binomial Probability Distribution (*Continued*)

$n = 16$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.185	.028	.003	.000	.000	.000	.000	.000	.000
1	.329	.113	.023	.003	.000	.000	.000	.000	.000
2	.275	.211	.073	.015	.002	.000	.000	.000	.000
3	.142	.246	.146	.047	.009	.001	.000	.000	.000
4	.051	.200	.204	.101	.028	.004	.000	.000	.000
5	.014	.120	.210	.162	.067	.014	.001	.000	.000
6	.003	.055	.165	.198	.122	.039	.006	.000	.000
7	.000	.020	.101	.189	.175	.084	.019	.001	.000
8	.000	.006	.049	.142	.196	.142	.049	.006	.000
9	.000	.001	.019	.084	.175	.189	.101	.020	.000
10	.000	.000	.006	.039	.122	.198	.165	.055	.003
11	.000	.000	.001	.014	.067	.162	.210	.120	.014
12	.000	.000	.000	.004	.028	.101	.204	.200	.051
13	.000	.000	.000	.001	.009	.047	.146	.246	.142
14	.000	.000	.000	.000	.002	.015	.073	.211	.275
15	.000	.000	.000	.000	.000	.003	.023	.113	.329
16	.000	.000	.000	.000	.000	.000	.003	.028	.185

$n = 17$

	Probability								
x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.167	.023	.002	.000	.000	.000	.000	.000	.000
1	.315	.096	.017	.002	.000	.000	.000	.000	.000
2	.280	.191	.058	.010	.001	.000	.000	.000	.000
3	.156	.239	.125	.034	.005	.000	.000	.000	.000
4	.060	.209	.187	.080	.018	.002	.000	.000	.000
5	.017	.136	.208	.138	.047	.008	.001	.000	.000
6	.004	.068	.178	.184	.094	.024	.003	.000	.000
7	.001	.027	.120	.193	.148	.057	.009	.000	.000
8	.000	.008	.064	.161	.185	.107	.028	.002	.000
9	.000	.002	.028	.107	.185	.161	.064	.008	.000
10	.000	.000	.009	.057	.148	.193	.120	.027	.001
11	.000	.000	.003	.024	.094	.184	.178	.068	.004
12	.000	.000	.001	.008	.047	.138	.208	.136	.017
13	.000	.000	.000	.002	.018	.080	.187	.209	.060
14	.000	.000	.000	.000	.005	.034	.125	.239	.156
15	.000	.000	.000	.000	.001	.010	.058	.191	.280
16	.000	.000	.000	.000	.000	.002	.017	.096	.315
17	.000	.000	.000	.000	.000	.000	.002	.023	.167

(*Continued*)

TABLE A.2

Binomial Probability Distribution (*Continued*)

$n = 18$

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
				Probability					
0	.150	.018	.002	.000	.000	.000	.000	.000	.000
1	.300	.081	.013	.001	.000	.000	.000	.000	.000
2	.284	.172	.046	.007	.001	.000	.000	.000	.000
3	.168	.230	.105	.025	.003	.000	.000	.000	.000
4	.070	.215	.168	.061	.012	.001	.000	.000	.000
5	.022	.151	.202	.115	.033	.004	.000	.000	.000
6	.005	.082	.187	.166	.071	.015	.001	.000	.000
7	.001	.035	.138	.189	.121	.037	.005	.000	.000
8	.000	.012	.081	.173	.167	.077	.015	.001	.000
9	.000	.003	.039	.128	.185	.128	.039	.003	.000
10	.000	.001	.015	.077	.167	.173	.081	.012	.000
11	.000	.000	.005	.037	.121	.189	.138	.035	.001
12	.000	.000	.001	.015	.071	.166	.187	.082	.005
13	.000	.000	.000	.004	.033	.115	.202	.151	.022
14	.000	.000	.000	.001	.012	.061	.168	.215	.070
15	.000	.000	.000	.000	.003	.025	.105	.230	.168
16	.000	.000	.000	.000	.001	.007	.046	.172	.284
17	.000	.000	.000	.000	.000	.001	.013	.081	.300
18	.000	.000	.000	.000	.000	.000	.002	.018	.150

$n = 19$

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
				Probability					
0	.135	.014	.001	.000	.000	.000	.000	.000	.000
1	.285	.068	.009	.001	.000	.000	.000	.000	.000
2	.285	.154	.036	.005	.000	.000	.000	.000	.000
3	.180	.218	.087	.017	.002	.000	.000	.000	.000
4	.080	.218	.149	.047	.007	.001	.000	.000	.000
5	.027	.164	.192	.093	.022	.002	.000	.000	.000
6	.007	.095	.192	.145	.052	.008	.001	.000	.000
7	.001	.044	.153	.180	.096	.024	.002	.000	.000
8	.000	.017	.098	.180	.144	.053	.008	.000	.000
9	.000	.005	.051	.146	.176	.098	.022	.001	.000
10	.000	.001	.022	.098	.176	.146	.051	.005	.000
11	.000	.000	.008	.053	.144	.180	.098	.017	.000
12	.000	.000	.002	.024	.096	.180	.153	.044	.001
13	.000	.000	.001	.008	.052	.145	.192	.095	.007
14	.000	.000	.000	.002	.022	.093	.192	.164	.027
15	.000	.000	.000	.001	.007	.047	.149	.218	.080
16	.000	.000	.000	.000	.002	.017	.087	.218	.180
17	.000	.000	.000	.000	.000	.005	.036	.154	.285
18	.000	.000	.000	.000	.000	.001	.009	.068	.285
19	.000	.000	.000	.000	.000	.000	.001	.014	.135

TABLE A.2

Binomial Probability Distribution (*Continued*)

$n = 20$

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.122	.012	.001	.000	.000	.000	.000	.000	.000
1	.270	.058	.007	.000	.000	.000	.000	.000	.000
2	.285	.137	.028	.003	.000	.000	.000	.000	.000
3	.190	.205	.072	.012	.001	.000	.000	.000	.000
4	.090	.218	.130	.035	.005	.000	.000	.000	.000
5	.032	.175	.179	.075	.015	.001	.000	.000	.000
6	.009	.109	.192	.124	.037	.005	.000	.000	.000
7	.002	.055	.164	.166	.074	.015	.001	.000	.000
8	.000	.022	.114	.180	.120	.035	.004	.000	.000
9	.000	.007	.065	.160	.160	.071	.012	.000	.000
10	.000	.002	.031	.117	.176	.117	.031	.002	.000
11	.000	.000	.012	.071	.160	.160	.065	.007	.000
12	.000	.000	.004	.035	.120	.180	.114	.022	.000
13	.000	.000	.001	.015	.074	.166	.164	.055	.002
14	.000	.000	.000	.005	.037	.124	.192	.109	.009
15	.000	.000	.000	.001	.015	.075	.179	.175	.032
16	.000	.000	.000	.000	.005	.035	.130	.218	.090
17	.000	.000	.000	.000	.001	.012	.072	.205	.190
18	.000	.000	.000	.000	.000	.003	.028	.137	.285
19	.000	.000	.000	.000	.000	.000	.007	.058	.270
20	.000	.000	.000	.000	.000	.000	.001	.012	.122

Probability (column headings .1 – .9)

$n = 25$

x	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.072	.004	.000	.000	.000	.000	.000	.000	.000
1	.199	.024	.001	.000	.000	.000	.000	.000	.000
2	.266	.071	.007	.000	.000	.000	.000	.000	.000
3	.226	.136	.024	.002	.000	.000	.000	.000	.000
4	.138	.187	.057	.007	.000	.000	.000	.000	.000
5	.065	.196	.103	.020	.002	.000	.000	.000	.000
6	.024	.163	.147	.044	.005	.000	.000	.000	.000
7	.007	.111	.171	.080	.014	.001	.000	.000	.000
8	.002	.062	.165	.120	.032	.003	.000	.000	.000
9	.000	.029	.134	.151	.061	.009	.000	.000	.000
10	.000	.012	.092	.161	.097	.021	.001	.000	.000
11	.000	.004	.054	.147	.133	.043	.004	.000	.000
12	.000	.001	.027	.114	.155	.076	.011	.000	.000
13	.000	.000	.011	.076	.155	.114	.027	.001	.000
14	.000	.000	.004	.043	.133	.147	.054	.004	.000
15	.000	.000	.001	.021	.097	.161	.092	.012	.000
16	.000	.000	.000	.009	.061	.151	.134	.029	.000
17	.000	.000	.000	.003	.032	.120	.165	.062	.002
18	.000	.000	.000	.001	.014	.080	.171	.111	.007
19	.000	.000	.000	.000	.005	.044	.147	.163	.024
20	.000	.000	.000	.000	.002	.020	.103	.196	.065
21	.000	.000	.000	.000	.000	.007	.057	.187	.138
22	.000	.000	.000	.000	.000	.002	.024	.136	.226
23	.000	.000	.000	.000	.000	.000	.007	.071	.266
24	.000	.000	.000	.000	.000	.000	.001	.024	.199
25	.000	.000	.000	.000	.000	.000	.000	.004	.072

Probability (column headings .1 – .9)

TABLE A.3

Poisson Probabilities

	λ									
x	.005	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.9950	.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
1	.0050	.0099	.0196	.0291	.0384	.0476	.0565	.0653	.0738	.0823
2	.0000	.0000	.0002	.0004	.0008	.0012	.0017	.0023	.0030	.0037
3	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001
x	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707
3	.0738	.0867	.0998	.1128	.1255	.1378	.1496	.1607	.1710	.1804
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034
8	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002
x	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1494
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	.1680
5	.0417	.0476	.0538	.0602	.0668	.0735	.0804	.0872	.0940	.1008
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

TABLE A.3

Poisson Probabilities (*Continued*)

	λ									
x	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**	**3.6**	**3.7**	**3.8**	**3.9**	**4.0**
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247	.0224	.0202	.0183
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915	.0850	.0789	.0733
2	.2165	.2087	.2008	.1929	.1850	.1771	.1692	.1615	.1539	.1465
3	.2237	.2226	.2209	.2186	.2158	.2125	.2087	.2046	.2001	.1954
4	.1733	.1781	.1823	.1858	.1888	.1912	.1931	.1944	.1951	.1954
5	.1075	.1140	.1203	.1264	.1322	.1377	.1429	.1477	.1522	.1563
6	.0555	.0608	.0662	.0716	.0771	.0826	.0881	.0936	.0989	.1042
7	.0246	.0278	.0312	.0348	.0385	.0425	.0466	.0508	.0551	.0595
8	.0095	.0111	.0129	.0148	.0169	.0191	.0215	.0241	.0269	.0298
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089	.0102	.0116	.0132
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033	.0039	.0045	.0053
11	.0003	.0004	.0005	.0006	.0007	.0009	.0011	.0013	.0016	.0019
12	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006
13	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
x	**4.1**	**4.2**	**4.3**	**4.4**	**4.5**	**4.6**	**4.7**	**4.8**	**4.9**	**5.0**
0	.0166	.0150	.0136	.0123	.0111	.0101	.0091	.0082	.0074	.0067
1	.0679	.0630	.0583	.0540	.0500	.0462	.0427	.0395	.0365	.0337
2	.1393	.1323	.1254	.1188	.1125	.1063	.1005	.0948	.0894	.0842
3	.1904	.1852	.1798	.1743	.1687	.1631	.1574	.1517	.1460	.1404
4	.1951	.1944	.1933	.1917	.1898	.1875	.1849	.1820	.1789	.1755
5	.1600	.1633	.1662	.1687	.1708	.1725	.1738	.1747	.1753	.1755
6	.1093	.1143	.1191	.1237	.1281	.1323	.1362	.1398	.1432	.1462
7	.0640	.0686	.0732	.0778	.0824	.0869	.0914	.0959	.1002	.1044
8	.0328	.0360	.0393	.0428	.0463	.0500	.0537	.0575	.0614	.0653
9	.0150	.0168	.0188	.0209	.0232	.0255	.0281	.0307	.0334	.0363
10	.0061	.0071	.0081	.0092	.0104	.0118	.0132	.0147	.0164	.0181
11	.0023	.0027	.0032	.0037	.0043	.0049	.0056	.0064	.0073	.0082
12	.0008	.0009	.0011	.0013	.0016	.0019	.0022	.0026	.0030	.0034
13	.0002	.0003	.0004	.0005	.0006	.0007	.0008	.0009	.0011	.0013
14	.0001	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005
15	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002

(*Continued*)

TABLE A.3

Poisson Probabilities (*Continued*)

	λ									
x	**5.1**	**5.2**	**5.3**	**5.4**	**5.5**	**5.6**	**5.7**	**5.8**	**5.9**	**6.0**
0	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027	.0025
1	.0311	.0287	.0265	.0244	.0225	.0207	.0191	.0176	.0162	.0149
2	.0793	.0746	.0701	.0659	.0618	.0580	.0544	.0509	.0477	.0446
3	.1348	.1293	.1239	.1185	.1133	.1082	.1033	.0985	.0938	.0892
4	.1719	.1681	.1641	.1600	.1558	.1515	.1472	.1428	.1383	.1339
5	.1753	.1748	.1740	.1728	.1714	.1697	.1678	.1656	.1632	.1606
6	.1490	.1515	.1537	.1555	.1571	.1584	.1594	.1601	.1605	.1606
7	.1086	.1125	.1163	.1200	.1234	.1267	.1298	.1326	.1353	.1377
8	.0692	.0731	.0771	.0810	.0849	.0887	.0925	.0962	.0998	.1033
9	.0392	.0423	.0454	.0486	.0519	.0552	.0586	.0620	.0654	.0688
10	.0200	.0220	.0241	.0262	.0285	.0309	.0334	.0359	.0386	.0413
11	.0093	.0104	.0116	.0129	.0143	.0157	.0173	.0190	.0207	.0225
12	.0039	.0045	.0051	.0058	.0065	.0073	.0082	.0092	.0102	.0113
13	.0015	.0018	.0021	.0024	.0028	.0032	.0036	.0041	.0046	.0052
14	.0006	.0007	.0008	.0009	.0011	.0013	.0015	.0017	.0019	.0022
15	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007	.0008	.0009
16	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003
17	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001
x	**6.1**	**6.2**	**6.3**	**6.4**	**6.5**	**6.6**	**6.7**	**6.8**	**6.9**	**7.0**
0	.0022	.0020	.0018	.0017	.0015	.0014	.0012	.0011	.0010	.0009
1	.0137	.0126	.0116	.0106	.0098	.0090	.0082	.0076	.0070	.0064
2	.0417	.0390	.0364	.0340	.0318	.0296	.0276	.0258	.0240	.0223
3	.0848	.0806	.0765	.0726	.0688	.0652	.0617	.0584	.0552	.0521
4	.1294	.1249	.1205	.1162	.1118	.1076	.1034	.0992	.0952	.0912
5	.1579	.1549	.1519	.1487	.1454	.1420	.1385	.1349	.1314	.1277
6	.1605	.1601	.1595	.1586	.1575	.1562	.1546	.1529	.1511	.1490
7	.1399	.1418	.1435	.1450	.1462	.1472	.1480	.1486	.1489	.1490
8	.1066	.1099	.1130	.1160	.1188	.1215	.1240	.1263	.1284	.1304
9	.0723	.0757	.0791	.0825	.0858	.0891	.0923	.0954	.0985	.1014
10	.0441	.0469	.0498	.0528	.0558	.0588	.0618	.0649	.0679	.0710
11	.0244	.0265	.0285	.0307	.0330	.0353	.0377	.0401	.0426	.0452
12	.0124	.0137	.0150	.0164	.0179	.0194	.0210	.0227	.0245	.0263
13	.0058	.0065	.0073	.0081	.0089	.0099	.0108	.0119	.0130	.0142
14	.0025	.0029	.0033	.0037	.0041	.0046	.0052	.0058	.0064	.0071
15	.0010	.0012	.0014	.0016	.0018	.0020	.0023	.0026	.0029	.0033
16	.0004	.0005	.0005	.0006	.0007	.0008	.0010	.0011	.0013	.0014
17	.0001	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0006
18	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002
19	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

TABLE A.3

Poisson Probabilities (*Continued*)

	λ									
x	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	.0008	.0007	.0007	.0006	.0006	.0005	.0005	.0004	.0004	.0003
1	.0059	.0054	.0049	.0045	.0041	.0038	.0035	.0032	.0029	.0027
2	.0208	.0194	.0180	.0167	.0156	.0145	.0134	.0125	.0116	.0107
3	.0492	.0464	.0438	.0413	.0389	.0366	.0345	.0324	.0305	.0286
4	.0874	.0836	.0799	.0764	.0729	.0696	.0663	.0632	.0602	.0573
5	.1241	.1204	.1167	.1130	.1094	.1057	.1021	.0986	.0951	.0916
6	.1468	.1445	.1420	.1394	.1367	.1339	.1311	.1282	.1252	.1221
7	.1489	.1486	.1481	.1474	.1465	.1454	.1442	.1428	.1413	.1396
8	.1321	.1337	.1351	.1363	.1373	.1381	.1388	.1392	.1395	.1396
9	.1042	.1070	.1096	.1121	.1144	.1167	.1187	.1207	.1224	.1241
10	.0740	.0770	.0800	.0829	.0858	.0887	.0914	.0941	.0967	.0993
11	.0478	.0504	.0531	.0558	.0585	.0613	.0640	.0667	.0695	.0722
12	.0283	.0303	.0323	.0344	.0366	.0388	.0411	.0434	.0457	.0481
13	.0154	.0168	.0181	.0196	.0211	.0227	.0243	.0260	.0278	.0296
14	.0078	.0086	.0095	.0104	.0113	.0123	.0134	.0145	.0157	.0169
15	.0037	.0041	.0046	.0051	.0057	.0062	.0069	.0075	.0083	.0090
16	.0016	.0019	.0021	.0024	.0026	.0030	.0033	.0037	.0041	.0045
17	.0007	.0008	.0009	.0010	.0012	.0013	.0015	.0017	.0019	.0021
18	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
19	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003	.0003	.0004
20	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002
21	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
x	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	.0003	.0003	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001
1	.0025	.0023	.0021	.0019	.0017	.0016	.0014	.0013	.0012	.0011
2	.0100	.0092	.0086	.0079	.0074	.0068	.0063	.0058	.0054	.0050
3	.0269	.0252	.0237	.0222	.0208	.0195	.0183	.0171	.0160	.0150
4	.0544	.0517	.0491	.0466	.0443	.0420	.0398	.0377	.0357	.0337
5	.0882	.0849	.0816	.0784	.0752	.0722	.0692	.0663	.0635	.0607
6	.1191	.1160	.1128	.1097	.1066	.1034	.1003	.0972	.0941	.0911
7	.1378	.1358	.1338	.1317	.1294	.1271	.1247	.1222	.1197	.1171
8	.1395	.1392	.1388	.1382	.1375	.1366	.1356	.1344	.1332	.1318
9	.1256	.1269	.1280	.1290	.1299	.1306	.1311	.1315	.1317	.1318
10	.1017	.1040	.1063	.1084	.1104	.1123	.1140	.1157	.1172	.1186
11	.0749	.0776	.0802	.0828	.0853	.0878	.0902	.0925	.0948	.0970
12	.0505	.0530	.0555	.0579	.0604	.0629	.0654	.0679	.0703	.0728
13	.0315	.0334	.0354	.0374	.0395	.0416	.0438	.0459	.0481	.0504
14	.0182	.0196	.0210	.0225	.0240	.0256	.0272	.0289	.0306	.0324
15	.0098	.0107	.0116	.0126	.0136	.0147	.0158	.0169	.0182	.0194
16	.0050	.0055	.0060	.0066	.0072	.0079	.0086	.0093	.0101	.0109
17	.0024	.0026	.0029	.0033	.0036	.0040	.0044	.0048	.0053	.0058
18	.0011	.0012	.0014	.0015	.0017	.0019	.0021	.0024	.0026	.0029
19	.0005	.0005	.0006	.0007	.0008	.0009	.0010	.0011	.0012	.0014
20	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0005	.0006
21	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0002	.0003
22	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001

(*Continued*)

TABLE A.3

Poisson Probabilities (*Continued*)

	λ									
x	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0
0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
1	.0010	.0009	.0009	.0008	.0007	.0007	.0006	.0005	.0005	.0005
2	.0046	.0043	.0040	.0037	.0034	.0031	.0029	.0027	.0025	.0023
3	.0140	.0131	.0123	.0115	.0107	.0100	.0093	.0087	.0081	.0076
4	.0319	.0302	.0285	.0269	.0254	.0240	.0226	.0213	.0201	.0189
5	.0581	.0555	.0530	.0506	.0483	.0460	.0439	.0418	.0398	.0378
6	.0881	.0851	.0822	.0793	.0764	.0736	.0709	.0682	.0656	.0631
7	.1145	.1118	.1091	.1064	.1037	.1010	.0982	.0955	.0928	.0901
8	.1302	.1286	.1269	.1251	.1232	.1212	.1191	.1170	.1148	.1126
9	.1317	.1315	.1311	.1306	.1300	.1293	.1284	.1274	.1263	.1251
10	.1198	.1210	.1219	.1228	.1235	.1241	.1245	.1249	.1250	.1251
11	.0991	.1012	.1031	.1049	.1067	.1083	.1098	.1112	.1125	.1137
12	.0752	.0776	.0799	.0822	.0844	.0866	.0888	.0908	.0928	.0948
13	.0526	.0549	.0572	.0594	.0617	.0640	.0662	.0685	.0707	.0729
14	.0342	.0361	.0380	.0399	.0419	.0439	.0459	.0479	.0500	.0521
15	.0208	.0221	.0235	.0250	.0265	.0281	.0297	.0313	.0330	.0347
16	.0118	.0127	.0137	.0147	.0157	.0168	.0180	.0192	.0204	.0217
17	.0063	.0069	.0075	.0081	.0088	.0095	.0103	.0111	.0119	.0128
18	.0032	.0035	.0039	.0042	.0046	.0051	.0055	.0060	.0065	.0071
19	.0015	.0017	.0019	.0021	.0023	.0026	.0028	.0031	.0034	.0037
20	.0007	.0008	.0009	.0010	.0011	.0012	.0014	.0015	.0017	.0019
21	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
22	.0001	.0001	.0002	.0002	.0002	.0002	.0003	.0003	.0004	.0004
23	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

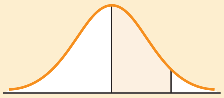
TABLE A.4

The e^{-x} Table

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0	1.0000	3.0	0.0498	6.0	0.00248	9.0	0.00012
0.1	0.9048	3.1	0.0450	6.1	0.00224	9.1	0.00011
0.2	0.8187	3.2	0.0408	6.2	0.00203	9.2	0.00010
0.3	0.7408	3.3	0.0369	6.3	0.00184	9.3	0.00009
0.4	0.6703	3.4	0.0334	6.4	0.00166	9.4	0.00008
0.5	0.6065	3.5	0.0302	6.5	0.00150	9.5	0.00007
0.6	0.5488	3.6	0.0273	6.6	0.00136	9.6	0.00007
0.7	0.4966	3.7	0.0247	6.7	0.00123	9.7	0.00006
0.8	0.4493	3.8	0.0224	6.8	0.00111	9.8	0.00006
0.9	0.4066	3.9	0.0202	6.9	0.00101	9.9	0.00005
1.0	0.3679	4.0	0.0183	7.0	0.00091	10.0	0.00005
1.1	0.3329	4.1	0.0166	7.1	0.00083		
1.2	0.3012	4.2	0.0150	7.2	0.00075		
1.3	0.2725	4.3	0.0136	7.3	0.00068		
1.4	0.2466	4.4	0.0123	7.4	0.00061		
1.5	0.2231	4.5	0.0111	7.5	0.00055		
1.6	0.2019	4.6	0.0101	7.6	0.00050		
1.7	0.1827	4.7	0.0091	7.7	0.00045		
1.8	0.1653	4.8	0.0082	7.8	0.00041		
1.9	0.1496	4.9	0.0074	7.9	0.00037		
2.0	0.1353	5.0	0.0067	8.0	0.00034		
2.1	0.1225	5.1	0.0061	8.1	0.00030		
2.2	0.1108	5.2	0.0055	8.2	0.00027		
2.3	0.1003	5.3	0.0050	8.3	0.00025		
2.4	0.0907	5.4	0.0045	8.4	0.00022		
2.5	0.0821	5.5	0.0041	8.5	0.00020		
2.6	0.0743	5.6	0.0037	8.6	0.00018		
2.7	0.0672	5.7	0.0033	8.7	0.00017		
2.8	0.0608	5.8	0.0030	8.8	0.00015		
2.9	0.0550	5.9	0.0027	8.9	0.00014		

TABLE A.5

Areas of the Standard Normal Distribution

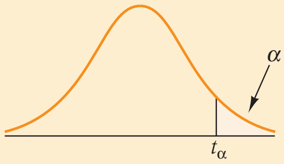


The entries in this table are the probabilities that a standard normal random variable is between 0 and z (the shaded area).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									
6.0	.499999999									

TABLE A.6

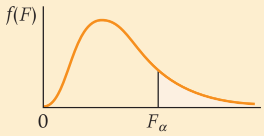
Critical Values from the *t* Distribution



	Values of α for one-tailed test and $\alpha/2$ for two-tailed test					
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2.000	2.390	2.660	3.232
70	1.294	1.667	1.994	2.381	2.648	3.211
80	1.292	1.664	1.990	2.374	2.639	3.195
90	1.291	1.662	1.987	2.368	2.632	3.183
100	1.290	1.660	1.984	2.364	2.626	3.174
150	1.287	1.655	1.976	2.351	2.609	3.145
200	1.286	1.653	1.972	2.345	2.601	3.131
∞	1.282	1.645	1.960	2.326	2.576	3.090

TABLE A.7

Percentage Points of the F Distribution



$\alpha = .10$

Numerator Degrees of Freedom (ν_1); Denominator Degrees of Freedom (ν_2)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .10$

Numerator Degrees of Freedom (ν_1); Denominator Degrees of Freedom (ν_2)

10	12	15	20	24	30	40	60	120	∞	ν_2
60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33	1
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49	2
5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13	3
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76	4
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10	5
2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72	6
2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47	7
2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29	8
2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16	9
2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06	10
2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97	11
2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90	12
2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85	13
2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80	14
2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76	15
2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72	16
2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69	17
1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66	18
1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63	19
1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61	20
1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59	21
1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57	22
1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55	23
1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53	24
1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52	25
1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50	26
1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49	27
1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48	28
1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47	29
1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46	30
1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38	40
1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29	60
1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19	120
1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00	∞

(*Continued*)

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .05$

Numerator Degrees of Freedom (v_1); Denominator Degrees of Freedom (v_2)

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .05$

Numerator Degrees of Freedom (ν_1)

10	12	15	20	24	30	40	60	120	∞	Denominator Degrees of Freedom (ν_2)
241.88	243.90	245.90	248.00	249.10	250.10	251.10	252.20	253.30	254.30	1
19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	2
8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	3
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	4
4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	5
4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	6
3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	7
3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	8
3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	9
2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	10
2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	11
2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	12
2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	13
2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	14
2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	15
2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	16
2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	17
2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	18
2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	19
2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	20
2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	21
2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	22
2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	23
2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	24
2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	25
2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	26
2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	27
2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	28
2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	29
2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	30
2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	40
1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	60
1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	120
1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	∞

(*Continued*)

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .025$

v_2 \ v_1 (Denominator Degrees of Freedom \ Numerator Degrees of Freedom)	1	2	3	4	5	6	7	8	9
1	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11

TABLE A.7

Percentage Points of the F Distribution (*Continued*)

$\alpha = .025$

Numerator Degrees of Freedom (v_1); Denominator Degrees of Freedom (v_2)

10	12	15	20	24	30	40	60	120	∞	v_2 / v_1
968.63	976.72	984.87	993.08	997.27	1001.40	1005.60	1009.79	1014.04	1018.00	1
39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50	2
14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90	3
8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26	4
6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02	5
5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85	6
4.76	4.67	4.57	4.47	4.41	4.36	4.31	4.25	4.20	4.14	7
4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67	8
3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33	9
3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08	10
3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88	11
3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72	12
3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60	13
3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49	14
3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40	15
2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32	16
2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25	17
2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19	18
2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13	19
2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09	20
2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04	21
2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00	22
2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97	23
2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94	24
2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91	25
2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88	26
2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85	27
2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83	28
2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81	29
2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79	30
2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64	40
2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48	60
2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31	120
2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00	∞

(*Continued*)

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .01$

Numerator Degrees of Freedom (ν_1); Denominator Degrees of Freedom (ν_2)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9
1	4052.18	4999.34	5403.53	5624.26	5763.96	5858.95	5928.33	5980.95	6022.40
2	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .01$

Numerator Degrees of Freedom (v_1); Denominator Degrees of Freedom (v_2)

10	12	15	20	24	30	40	60	120	∞	v_2
6055.93	6106.68	6156.97	6208.66	6234.27	6260.35	6286.43	6312.97	6339.51	6366.00	1
99.40	99.42	99.43	99.45	99.46	99.47	99.48	99.48	99.49	99.50	2
27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	3
14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	4
10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	5
7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	6
6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	7
5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	8
5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	9
4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	10
4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	11
4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	12
4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	13
3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00	14
3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	15
3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	16
3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65	17
3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	18
3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	19
3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	20
3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	21
3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	22
3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26	23
3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21	24
3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	25
3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13	26
3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10	27
3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06	28
3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03	29
2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	30
2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80	40
2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	60
2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	120
2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00	∞

(*Continued*)

TABLE A.7

Percentage Points of the *F* Distribution (*Continued*)

$\alpha = .005$

Numerator Degrees of Freedom (ν_1); Denominator Degrees of Freedom (ν_2)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9
1	16212.46	19997.36	21614.13	22500.75	23055.82	23439.53	23715.20	23923.81	24091.45
2	198.50	199.01	199.16	199.24	199.30	199.33	199.36	199.38	199.39
3	55.55	49.80	47.47	46.20	45.39	44.84	44.43	44.13	43.88
4	31.33	26.28	24.26	23.15	22.46	21.98	21.62	21.35	21.14
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
11	12.23	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
13	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94
14	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54
16	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38
17	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25
18	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14
19	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64
26	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60
27	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56
28	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52
29	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62

TABLE A.7

Percentage Points of the F Distribution (*Continued*)

$\alpha = .005$

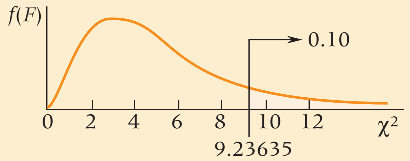
Numerator Degrees of Freedom (v_1)

v_2 \ v_1	10	12	15	20	24	30	40	60	120	∞
1	24221.84	24426.73	24631.62	24836.51	24937.09	25041.40	25145.71	25253.74	25358.05	25465.00
2	199.39	199.42	199.43	199.45	199.45	199.48	199.48	199.48	199.49	199.50
3	43.68	43.39	43.08	42.78	42.62	42.47	42.31	42.15	41.99	41.83
4	20.97	20.70	20.44	20.17	20.03	19.89	19.75	19.61	19.47	19.32
5	13.62	13.38	13.15	12.90	12.78	12.66	12.53	12.40	12.27	12.14
6	10.25	10.03	9.81	9.59	9.47	9.36	9.24	9.12	9.00	8.88
7	8.38	8.18	7.97	7.75	7.64	7.53	7.42	7.31	7.19	7.08
8	7.21	7.01	6.81	6.61	6.50	6.40	6.29	6.18	6.06	5.95
9	6.42	6.23	6.03	5.83	5.73	5.62	5.52	5.41	5.30	5.19
10	5.85	5.66	5.47	5.27	5.17	5.07	4.97	4.86	4.75	4.64
11	5.42	5.24	5.05	4.86	4.76	4.65	4.55	4.45	4.34	4.23
12	5.09	4.91	4.72	4.53	4.43	4.33	4.23	4.12	4.01	3.90
13	4.82	4.64	4.46	4.27	4.17	4.07	3.97	3.87	3.76	3.65
14	4.60	4.43	4.25	4.06	3.96	3.86	3.76	3.66	3.55	3.44
15	4.42	4.25	4.07	3.88	3.79	3.69	3.59	3.48	3.37	3.26
16	4.27	4.10	3.92	3.73	3.64	3.54	3.44	3.33	3.22	3.11
17	4.14	3.97	3.79	3.61	3.51	3.41	3.31	3.21	3.10	2.98
18	4.03	3.86	3.68	3.50	3.40	3.30	3.20	3.10	2.99	2.87
19	3.93	3.76	3.59	3.40	3.31	3.21	3.11	3.00	2.89	2.78
20	3.85	3.68	3.50	3.32	3.22	3.12	3.02	2.92	2.81	2.69
21	3.77	3.60	3.43	3.24	3.15	3.05	2.95	2.84	2.73	2.61
22	3.70	3.54	3.36	3.18	3.08	2.98	2.88	2.77	2.66	2.55
23	3.64	3.47	3.30	3.12	3.02	2.92	2.82	2.71	2.60	2.48
24	3.59	3.42	3.25	3.06	2.97	2.87	2.77	2.66	2.55	2.43
25	3.54	3.37	3.20	3.01	2.92	2.82	2.72	2.61	2.50	2.38
26	3.49	3.33	3.15	2.97	2.87	2.77	2.67	2.56	2.45	2.33
27	3.45	3.28	3.11	2.93	2.83	2.73	2.63	2.52	2.41	2.29
28	3.41	3.25	3.07	2.89	2.79	2.69	2.59	2.48	2.37	2.25
29	3.38	3.21	3.04	2.86	2.76	2.66	2.56	2.45	2.33	2.21
30	3.34	3.18	3.01	2.82	2.73	2.63	2.52	2.42	2.30	2.18
40	3.12	2.95	2.78	2.60	2.50	2.40	2.30	2.18	2.06	1.93
60	2.90	2.74	2.57	2.39	2.29	2.19	2.08	1.96	1.83	1.69
120	2.71	2.54	2.37	2.19	2.09	1.98	1.87	1.75	1.61	1.43
∞	2.52	2.36	2.19	2.00	1.90	1.79	1.67	1.53	1.36	1.00

TABLE A.8

The Chi-Square Table

Values of χ^2 for Selected Probabilities



Example: df (Number of degrees of freedom) = 5, the tail above $\chi^2 = 9.23635$ represents 0.10 or 10% of area under the curve.

Degrees of Freedom	Area in Upper Tail .995	.99	.975	.95	.9	.1	.05	.025	.01	.005
1	0.0000393	0.0001571	0.0009821	0.0039322	0.0157907	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.010025	0.020100	0.050636	0.102586	0.210721	4.6052	5.9915	7.3778	9.2104	10.5965
3	0.07172	0.11483	0.21579	0.35185	0.58438	6.2514	7.8147	9.3484	11.3449	12.8381
4	0.20698	0.29711	0.48442	0.71072	1.06362	7.7794	9.4877	11.1433	13.2767	14.8602
5	0.41175	0.55430	0.83121	1.14548	1.61031	9.2363	11.0705	12.8325	15.0863	16.7496
6	0.67573	0.87208	1.23734	1.63538	2.20413	10.6446	12.5916	14.4494	16.8119	18.5475
7	0.98925	1.23903	1.68986	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.34440	1.64651	2.17972	2.73263	3.48954	13.3616	15.5073	17.5345	20.0902	21.9549
9	1.73491	2.08789	2.70039	3.32512	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55820	3.24696	3.94030	4.86518	15.9872	18.3070	20.4832	23.2093	25.1881
11	2.60320	3.05350	3.81574	4.57481	5.57779	17.2750	19.6752	21.9200	24.7250	26.7569
12	3.07379	3.57055	4.40378	5.22603	6.30380	18.5493	21.0261	23.3367	26.2170	28.2997
13	3.56504	4.10690	5.00874	5.89186	7.04150	19.8119	22.3620	24.7356	27.6882	29.8193
14	4.07466	4.66042	5.62872	6.57063	7.78954	21.0641	23.6848	26.1189	29.1412	31.3194
15	4.60087	5.22936	6.26212	7.26093	8.54675	22.3071	24.9958	27.4884	30.5780	32.8015
16	5.14216	5.81220	6.90766	7.96164	9.31224	23.5418	26.2962	28.8453	31.9999	34.2671
17	5.69727	6.40774	7.56418	8.67175	10.08518	24.7690	27.5871	30.1910	33.4087	35.7184
18	6.26477	7.01490	8.23074	9.39045	10.86494	25.9894	28.8693	31.5264	34.8052	37.1564
19	6.84392	7.63270	8.90651	10.11701	11.65091	27.2036	30.1435	32.8523	36.1908	38.5821
20	7.43381	8.26037	9.59077	10.85080	12.44260	28.4120	31.4104	34.1696	37.5663	39.9969
21	8.03360	8.89717	10.28291	11.59132	13.23960	29.6151	32.6706	35.4789	38.9322	41.4009
22	8.64268	9.54249	10.98233	12.33801	14.04149	30.8133	33.9245	36.7807	40.2894	42.7957
23	9.26038	10.19569	11.68853	13.09051	14.84795	32.0069	35.1725	38.0756	41.6383	44.1814
24	9.88620	10.85635	12.40115	13.84842	15.65868	33.1962	36.4150	39.3641	42.9798	45.5584
25	10.51965	11.52395	13.11971	14.61140	16.47341	34.3816	37.6525	40.6465	44.3140	46.9280
26	11.16022	12.19818	13.84388	15.37916	17.29188	35.5632	38.8851	41.9231	45.6416	48.2898
27	11.80765	12.87847	14.57337	16.15139	18.11389	36.7412	40.1133	43.1945	46.9628	49.6450
28	12.46128	13.56467	15.30785	16.92788	18.93924	37.9159	41.3372	44.4608	48.2782	50.9936
29	13.12107	14.25641	16.04705	17.70838	19.76774	39.0875	42.5569	45.7223	49.5878	52.3355
30	13.78668	14.95346	16.79076	18.49267	20.59924	40.2560	43.7730	46.9792	50.8922	53.6719
40	20.70658	22.16420	24.43306	26.50930	29.05052	51.8050	55.7585	59.3417	63.6908	66.7660
50	27.99082	29.70673	32.35738	34.76424	37.68864	63.1671	67.5048	71.4202	76.1538	79.4898
60	35.53440	37.48480	40.48171	43.18797	46.45888	74.3970	79.0820	83.2977	88.3794	91.9518
70	43.27531	45.44170	48.75754	51.73926	55.32894	85.5270	90.5313	95.0231	100.4251	104.2148
80	51.17193	53.53998	57.15315	60.39146	64.27784	96.5782	101.8795	106.6285	112.3288	116.3209
90	59.19633	61.75402	65.64659	69.12602	73.29108	107.5650	113.1452	118.1359	124.1162	128.2987
100	67.32753	70.06500	74.22188	77.92944	82.35813	118.4980	124.3221	129.5613	135.8069	140.1697

TABLE A.9

Critical Values for the Durbin-Watson Test

Entries in the table give the critical values for a one-tailed Durbin-Watson test for autocorrelation. For a two-tailed test, the level of significance is doubled.

Significant Points of d_L and d_U: $\alpha = .05$
Number of Independent Variables

k	1		2		3		4		5	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

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(*Continued*)

TABLE A.9

Critical Values for the Durbin-Watson Test (*Continued*)

Significant Points of d_L and d_U: $\alpha = .01$
Number of Independent Variables

k	1		2		3		4		5	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	0.81	1.07	0.70	1.25	0.59	1.46	0.49	1.70	0.39	1.96
16	0.84	1.09	0.74	1.25	0.63	1.44	0.53	1.66	0.44	1.90
17	0.87	1.10	0.77	1.25	0.67	1.43	0.57	1.63	0.48	1.85
18	0.90	1.12	0.80	1.26	0.71	1.42	0.61	1.60	0.52	1.80
19	0.93	1.13	0.83	1.26	0.74	1.41	0.65	1.58	0.56	1.77
20	0.95	1.15	0.86	1.27	0.77	1.41	0.68	1.57	0.60	1.74
21	0.97	1.16	0.89	1.27	0.80	1.41	0.72	1.55	0.63	1.71
22	1.00	1.17	0.91	1.28	0.83	1.40	0.75	1.54	0.66	1.69
23	1.02	1.19	0.94	1.29	0.86	1.40	0.77	1.53	0.70	1.67
24	1.04	1.20	0.96	1.30	0.88	1.41	0.80	1.53	0.72	1.66
25	1.05	1.21	0.98	1.30	0.90	1.41	0.83	1.52	0.75	1.65
26	1.07	1.22	1.00	1.31	0.93	1.41	0.85	1.52	0.78	1.64
27	1.09	1.23	1.02	1.32	0.95	1.41	0.88	1.51	0.81	1.63
28	1.10	1.24	1.04	1.32	0.97	1.41	0.90	1.51	0.83	1.62
29	1.12	1.25	1.05	1.33	0.99	1.42	0.92	1.51	0.85	1.61
30	1.13	1.26	1.07	1.34	1.01	1.42	0.94	1.51	0.88	1.61
31	1.15	1.27	1.08	1.34	1.02	1.42	0.96	1.51	0.90	1.60
32	1.16	1.28	1.10	1.35	1.04	1.43	0.98	1.51	0.92	1.60
33	1.17	1.29	1.11	1.36	1.05	1.43	1.00	1.51	0.94	1.59
34	1.18	1.30	1.13	1.36	1.07	1.43	1.01	1.51	0.95	1.59
35	1.19	1.31	1.14	1.37	1.08	1.44	1.03	1.51	0.97	1.59
36	1.21	1.32	1.15	1.38	1.10	1.44	1.04	1.51	0.99	1.59
37	1.22	1.32	1.16	1.38	1.11	1.45	1.06	1.51	1.00	1.59
38	1.23	1.33	1.18	1.39	1.12	1.45	1.07	1.52	1.02	1.58
39	1.24	1.34	1.19	1.39	1.14	1.45	1.09	1.52	1.03	1.58
40	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
45	1.29	1.38	1.24	1.42	1.20	1.48	1.16	1.53	1.11	1.58
50	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
55	1.36	1.43	1.32	1.47	1.28	1.51	1.25	1.55	1.21	1.59
60	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
65	1.41	1.47	1.38	1.50	1.35	1.53	1.31	1.57	1.28	1.61
70	1.43	1.49	1.40	1.52	1.37	1.55	1.34	1.58	1.31	1.61
75	1.45	1.50	1.42	1.53	1.39	1.56	1.37	1.59	1.34	1.62
80	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
85	1.48	1.53	1.46	1.55	1.43	1.58	1.41	1.60	1.39	1.63
90	1.50	1.54	1.47	1.56	1.45	1.59	1.43	1.61	1.41	1.64
95	1.51	1.55	1.49	1.57	1.47	1.60	1.45	1.62	1.42	1.64
100	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

TABLE A.10

Critical Values of the Studentized Range (q) Distribution

$\alpha = .05$

Degrees of Freedom	Number of Populations																		
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	18.0	27.0	32.8	37.1	40.4	43.1	45.4	47.4	49.1	50.6	52.0	53.2	54.3	55.4	56.3	57.2	58.0	58.8	59.6
2	6.08	8.33	9.80	10.9	11.7	12.4	13.0	13.5	14.0	14.4	14.7	15.1	15.4	15.7	15.9	16.1	16.4	16.6	16.8
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.2	10.3	10.5	10.7	10.8	11.0	11.1	11.2
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.10	7.17
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.19	6.27	6.34	6.40	6.47
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.98	6.06	6.13	6.20	6.27	6.33
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	5.99	6.05	6.11
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.71	5.79	5.85	5.91	5.97	6.03
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.57	5.65	5.72	5.78	5.85	5.90	5.96
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.54	5.61	5.67	5.73	5.79	5.84
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.31	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.49	5.55	5.59
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.47
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82	4.90	4.98	5.04	5.11	5.16	5.22	5.27	5.31	5.36
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

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(*Continued*)

TABLE A.10

Critical Values of the Studentized Range (q) Distribution (*Continued*)

$\alpha = .01$

Degrees of Freedom	Number of Populations 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	90.0	135.	164.	186.	202.	216.	227.	237.	246.	253.	260.	266.	272.	277.	282.	286.	290.	294.	298.
2	14.0	19.0	22.3	24.7	26.6	28.2	29.5	30.7	31.7	32.6	33.4	34.1	34.8	35.4	36.0	36.5	37.0	37.5	37.9
3	8.26	10.6	12.2	13.3	14.2	15.0	15.6	16.2	16.7	17.1	17.5	17.9	18.2	18.5	18.8	19.1	19.3	19.5	19.8
4	6.51	8.12	9.17	9.96	10.6	11.1	11.5	11.9	12.3	12.6	12.8	13.1	13.3	13.5	13.7	13.9	14.1	14.2	14.4
5	5.70	6.97	7.80	8.42	8.91	9.32	9.67	9.97	10.2	10.5	10.7	10.9	11.1	11.2	11.4	11.6	11.7	11.8	11.9
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.49	9.65	9.81	9.95	10.1	10.2	10.3	10.4	10.5
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71	8.86	9.00	9.12	9.24	9.35	9.46	9.55	9.65
8	4.74	5.63	6.20	6.63	6.96	7.24	7.47	7.68	7.87	8.03	8.18	8.31	8.44	8.55	8.66	8.76	8.85	8.94	9.03
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.32	7.49	7.65	7.78	7.91	8.03	8.13	8.23	8.32	8.41	8.49	8.57
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.48	7.60	7.71	7.81	7.91	7.99	8.07	8.15	8.22
11	4.39	5.14	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13	7.25	7.36	7.46	7.56	7.65	7.73	7.81	7.88	7.95
12	4.32	5.04	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06	7.17	7.26	7.36	7.44	7.52	7.59	7.66	7.73
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90	7.01	7.10	7.19	7.27	7.34	7.42	7.48	7.55
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77	6.87	6.96	7.05	7.12	7.20	7.27	7.33	7.39
15	4.17	4.83	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55	6.66	6.76	6.84	6.93	7.00	7.07	7.14	7.20	7.26
16	4.13	4.78	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56	6.66	6.74	6.82	6.90	6.97	7.03	7.09	7.15
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38	6.48	6.57	6.66	6.73	6.80	6.87	6.94	7.00	7.05
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41	6.50	6.58	6.65	6.72	6.79	6.85	6.91	6.96
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25	6.34	6.43	6.51	6.58	6.65	6.72	6.78	6.84	6.89
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.29	6.37	6.45	6.52	6.59	6.65	6.71	6.76	6.82
24	3.96	4.54	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11	6.19	6.26	6.33	6.39	6.45	6.51	6.56	6.61
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85	5.93	6.01	6.08	6.14	6.20	6.26	6.31	6.36	6.41
40	3.82	4.37	4.70	4.93	5.11	5.27	5.39	5.50	5.60	5.69	5.77	5.84	5.90	5.96	6.02	6.07	6.12	6.17	6.21
60	3.76	4.28	4.60	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60	5.67	5.73	5.79	5.84	5.89	5.93	5.98	6.02
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.38	5.44	5.51	5.56	5.61	5.66	5.71	5.75	5.79	5.83
∞	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29	5.35	5.40	5.45	5.49	5.54	5.57	5.61	5.65

TABLE A.11

Critical Values of R for the Runs Test: Lower Tail

n_1 \ n_2									$\alpha = .025$										
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2	2	2	2	2	2	2	2	2
3					2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
4				2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
5			2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
6		2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
7		2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
8		2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
9		2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
10		2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
11		2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14

Source: Adapted from F. S. Swed and C. Eisenhart, *Ann. Math. Statist.*, vol. 14, 1943, pp. 83–86.

TABLE A.12

Critical Values of R for the Runs Test: Upper Tail

n_1 \ n_2									$\alpha = .025$										
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2																			
3																			
4				9	9														
5			9	10	10	11	11												
6			9	10	11	12	12	13	13	13	13								
7				11	12	13	13	14	14	14	14	15	15	15					
8				11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9					13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10					13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11					13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12					13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13						15	16	17	18	19	19	20	20	21	21	22	22	23	23
14						15	16	17	18	19	20	20	21	22	22	23	23	23	24
15						15	16	18	18	19	20	21	22	22	23	23	24	24	25
16							17	18	19	20	21	21	22	23	23	24	25	25	25
17							17	18	19	20	21	22	23	23	24	25	25	26	26
18							17	18	19	20	21	22	23	24	25	25	26	26	27
19							17	18	20	21	22	23	23	24	25	26	26	27	27
20							17	18	20	21	22	23	24	25	25	26	27	27	28

TABLE A.13

p-Values for Mann-Whitney *U* Statistic Small Samples ($n_1 \le n_2$)

$n_2 = 3$	U_0	n_1 = 1	2	3
	0	.25	.10	.05
	1	.50	.20	.10
	2		.40	.20
	3		.60	.35
	4			.50

$n_2 = 4$	U_0	n_1 = 1	2	3	4
	0	.2000	.0667	.0286	.0143
	1	.4000	.1333	.0571	.0286
	2	.6000	.2667	.1143	.0571
	3		.4000	.2000	.1000
	4		.6000	.3143	.1714
	5			.4286	.2429
	6			.5714	.3429
	7				.4429
	8				.5571

$n_2 = 5$	U_0	n_1 = 1	2	3	4	5
	0	.1667	.0476	.0179	.0079	.0040
	1	.3333	.0952	.0357	.0159	.0079
	2	.5000	.1905	.0714	.0317	.0159
	3		.2857	.1250	.0556	.0278
	4		.4286	.1964	.0952	.0476
	5		.5714	.2857	.1429	.0754
	6			.3929	.2063	.1111
	7			.5000	.2778	.1548
	8				.3651	.2103
	9				.4524	.2738
	10				.5476	.3452
	11					.4206
	12					.5000

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TABLE A.13

p-Values for Mann-Whitney *U* Statistic Small Samples ($n_1 \leq n_2$) (*Continued*)

		n_1					
$n_2 = 6$	U_0	1	2	3	4	5	6
	0	.1429	.0357	.0119	.0048	.0022	.0011
	1	.2857	.0714	.0238	.0095	.0043	.0022
	2	.4286	.1429	.0476	.0190	.0087	.0043
	3	.5714	.2143	.0833	.0333	.0152	.0076
	4		.3214	.1310	.0571	.0260	.0130
	5		.4286	.1905	.0857	.0411	.0206
	6		.5714	.2738	.1286	.0628	.0325
	7			.3571	.1762	.0887	.0465
	8			.4524	.2381	.1234	.0660
	9			.5476	.3048	.1645	.0898
	10				.3810	.2143	.1201
	11				.4571	.2684	.1548
	12				.5429	.3312	.1970
	13					.3961	.2424
	14					.4654	.2944
	15					.5346	.3496
	16						.4091
	17						.4686
	18						.5314

		n_1						
$n_2 = 7$	U_0	1	2	3	4	5	6	7
	0	.1250	.0278	.0083	.0030	.0013	.0006	.0003
	1	.2500	.0556	.0167	.0061	.0025	.0012	.0006
	2	.3750	.1111	.0333	.0121	.0051	.0023	.0012
	3	.5000	.1667	.0583	.0212	.0088	.0041	.0020
	4		.2500	.0917	.0364	.0152	.0070	.0035
	5		.3333	.1333	.0545	.0240	.0111	.0055
	6		.4444	.1917	.0818	.0366	.0175	.0087
	7		.5556	.2583	.1152	.0530	.0256	.0131
	8			.3333	.1576	.0745	.0367	.0189
	9			.4167	.2061	.1010	.0507	.0265
	10			.5000	.2636	.1338	.0688	.0364
	11				.3242	.1717	.0903	.0487
	12				.3939	.2159	.1171	.0641
	13				.4636	.2652	.1474	.0825
	14				.5364	.3194	.1830	.1043
	15					.3775	.2226	.1297
	16					.4381	.2669	.1588
	17					.5000	.3141	.1914
	18						.3654	.2279
	19						.4178	.2675
	20						.4726	.3100
	21						.5274	.3552
	22							.4024
	23							.4508
	24							.5000

(*Continued*)

TABLE A.13

p-Values for Mann-Whitney *U* Statistic Small Samples ($n_1 \leq n_2$) (*Continued*)

		n_1							
$n_2 = 8$	U_0	1	2	3	4	5	6	7	8
	0	.1111	.0222	.0061	.0020	.0008	.0003	.0002	.0001
	1	.2222	.0444	.0121	.0040	.0016	.0007	.0003	.0002
	2	.3333	.0889	.0242	.0081	.0031	.0013	.0006	.0003
	3	.4444	.1333	.0424	.0141	.0054	.0023	.0011	.0005
	4	.5556	.2000	.0667	.0242	.0093	.0040	.0019	.0009
	5		.2667	.0970	.0364	.0148	.0063	.0030	.0015
	6		.3556	.1394	.0545	.0225	.0100	.0047	.0023
	7		.4444	.1879	.0768	.0326	.0147	.0070	.0035
	8		.5556	.2485	.1071	.0466	.0213	.0103	.0052
	9			.3152	.1414	.0637	.0296	.0145	.0074
	10			.3879	.1838	.0855	.0406	.0200	.0103
	11			.4606	.2303	.1111	.0539	.0270	.0141
	12			.5394	.2848	.1422	.0709	.0361	.0190
	13				.3414	.1772	.0906	.0469	.0249
	14				.4040	.2176	.1142	.0603	.0325
	15				.4667	.2618	.1412	.0760	.0415
	16				.5333	.3108	.1725	.0946	.0524
	17					.3621	.2068	.1159	.0652
	18					.4165	.2454	.1405	.0803
	19					.4716	.2864	.1678	.0974
	20					.5284	.3310	.1984	.1172
	21						.3773	.2317	.1393
	22						.4259	.2679	.1641
	23						.4749	.3063	.1911
	24						.5251	.3472	.2209
	25							.3894	.2527
	26							.4333	.2869
	27							.4775	.3227
	28							.5225	.3605
	29								.3992
	30								.4392
	31								.4796
	32								.5204

TABLE A.13

p-Values for Mann-Whitney *U* Statistic Small Samples ($n_1 \leq n_2$) (*Continued*)

		n_1								
$n_2 = 9$	U_0	1	2	3	4	5	6	7	8	9
	0	.1000	.0182	.0045	.0014	.0005	.0002	.0001	.0000	.0000
	1	.2000	.0364	.0091	.0028	.0010	.0004	.0002	.0001	.0000
	2	.3000	.0727	.0182	.0056	.0020	.0008	.0003	.0002	.0001
	3	.4000	.1091	.0318	.0098	.0035	.0014	.0006	.0003	.0001
	4	.5000	.1636	.0500	.0168	.0060	.0024	.0010	.0005	.0002
	5		.2182	.0727	.0252	.0095	.0038	.0017	.0008	.0004
	6		.2909	.1045	.0378	.0145	.0060	.0026	.0012	.0006
	7		.3636	.1409	.0531	.0210	.0088	.0039	.0019	.0009
	8		.4545	.1864	.0741	.0300	.0128	.0058	.0028	.0014
	9		.5455	.2409	.0993	.0415	.0180	.0082	.0039	.0020
	10			.3000	.1301	.0559	.0248	.0115	.0056	.0028
	11			.3636	.1650	.0734	.0332	.0156	.0076	.0039
	12			.4318	.2070	.0949	.0440	.0209	.0103	.0053
	13			.5000	.2517	.1199	.0567	.0274	.0137	.0071
	14				.3021	.1489	.0723	.0356	.0180	.0094
	15				.3552	.1818	.0905	.0454	.0232	.0122
	16				.4126	.2188	.1119	.0571	.0296	.0157
	17				.4699	.2592	.1361	.0708	.0372	.0200
	18				.5301	.3032	.1638	.0869	.0464	.0252
	19					.3497	.1942	.1052	.0570	.0313
	20					.3986	.2280	.1261	.0694	.0385
	21					.4491	.2643	.1496	.0836	.0470
	22					.5000	.3035	.1755	.0998	.0567
	23						.3445	.2039	.1179	.0680
	24						.3878	.2349	.1383	.0807
	25						.4320	.2680	.1606	.0951
	26						.4773	.3032	.1852	.1112
	27						.5227	.3403	.2117	.1290
	28							.3788	.2404	.1487
	29							.4185	.2707	.1701
	30							.4591	.3029	.1933
	31							.5000	.3365	.2181
	32								.3715	.2447
	33								.4074	.2729
	34								.4442	.3024
	35								.4813	.3332
	36								.5187	.3652
	37									.3981
	38									.4317
	39									.4657
	40									.5000

(*Continued*)

TABLE A.13

p-Values for Mann-Whitney *U* Statistic Small Samples ($n_1 \leq n_2$) (*Continued*)

$n_2 = 10$ U_0	n_1 = 1	2	3	4	5	6	7	8	9	10
0	.0909	.0152	.0035	.0010	.0003	.0001	.0001	.0000	.0000	.0000
1	.1818	.0303	.0070	.0020	.0007	.0002	.0001	.0000	.0000	.0000
2	.2727	.0606	.0140	.0040	.0013	.0005	.0002	.0001	.0000	.0000
3	.3636	.0909	.0245	.0070	.0023	.0009	.0004	.0002	.0001	.0000
4	.4545	.1364	.0385	.0120	.0040	.0015	.0006	.0003	.0001	.0001
5	.5455	.1818	.0559	.0180	.0063	.0024	.0010	.0004	.0002	.0001
6		.2424	.0804	.0270	.0097	.0037	.0015	.0007	.0003	.0002
7		.3030	.1084	.0380	.0140	.0055	.0023	.0010	.0005	.0002
8		.3788	.1434	.0529	.0200	.0080	.0034	.0015	.0007	.0004
9		.4545	.1853	.0709	.0276	.0112	.0048	.0022	.0011	.0005
10		.5455	.2343	.0939	.0376	.0156	.0068	.0031	.0015	.0008
11			.2867	.1199	.0496	.0210	.0093	.0043	.0021	.0010
12			.3462	.1518	.0646	.0280	.0125	.0058	.0028	.0014
13			.4056	.1868	.0823	.0363	.0165	.0078	.0038	.0019
14			.4685	.2268	.1032	.0467	.0215	.0103	.0051	.0026
15			.5315	.2697	.1272	.0589	.0277	.0133	.0066	.0034
16				.3177	.1548	.0736	.0351	.0171	.0086	.0045
17				.3666	.1855	.0903	.0439	.0217	.0110	.0057
18				.4196	.2198	.1099	.0544	.0273	.0140	.0073
19				.4725	.2567	.1317	.0665	.0338	.0175	.0093
20				.5275	.2970	.1566	.0806	.0416	.0217	.0116
21					.3393	.1838	.0966	.0506	.0267	.0144
22					.3839	.2139	.1148	.0610	.0326	.0177
23					.4296	.2461	.1349	.0729	.0394	.0216
24					.4765	.2811	.1574	.0864	.0474	.0262
25					.5235	.3177	.1819	.1015	.0564	.0315
26						.3564	.2087	.1185	.0667	.0376
27						.3962	.2374	.1371	.0782	.0446
28						.4374	.2681	.1577	.0912	.0526
29						.4789	.3004	.1800	.1055	.0615
30						.5211	.3345	.2041	.1214	.0716
31							.3698	.2299	.1388	.0827
32							.4063	.2574	.1577	.0952
33							.4434	.2863	.1781	.1088
34							.4811	.3167	.2001	.1237
35							.5189	.3482	.2235	.1399
36								.3809	.2483	.1575
37								.4143	.2745	.1763
38								.4484	.3019	.1965
39								.4827	.3304	.2179
40								.5173	.3598	.2406
41									.3901	.2644
42									.4211	.2894
43									.4524	.3153
44									.4841	.3421
45									.5159	.3697
46										.3980
47										.4267
48										.4559
49										.4853
50										.5147

TABLE A.14

Critical Values of T for the Wilcoxon Matched-Pairs Signed Rank Test (Small Samples)

1-SIDED	2-SIDED	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$\alpha = .05$	$\alpha = .10$	1	2	4	6	8	11
$\alpha = .025$	$\alpha = .05$		1	2	4	6	8
$\alpha = .01$	$\alpha = .02$			0	2	3	5
$\alpha = .005$	$\alpha = .01$				0	2	3

1-SIDED	2-SIDED	$n = 11$	$n = 12$	$n = 13$	$n = 14$	$n = 15$	$n = 16$
$\alpha = .05$	$\alpha = .10$	14	17	21	26	30	36
$\alpha = .025$	$\alpha = .05$	11	14	17	21	25	30
$\alpha = .01$	$\alpha = .02$	7	10	13	16	20	24
$\alpha = .005$	$\alpha = .01$	5	7	10	13	16	19

1-SIDED	2-SIDED	$n = 17$	$n = 18$	$n = 19$	$n = 20$	$n = 21$	$n = 22$
$\alpha = .05$	$\alpha = .10$	41	47	54	60	68	75
$\alpha = .025$	$\alpha = .05$	35	40	46	52	59	66
$\alpha = .01$	$\alpha = .02$	28	33	38	43	49	56
$\alpha = .005$	$\alpha = .01$	23	28	32	37	43	49

1-SIDED	2-SIDED	$n = 23$	$n = 24$	$n = 25$	$n = 26$	$n = 27$	$n = 28$
$\alpha = .05$	$\alpha = .10$	83	92	101	110	120	130
$\alpha = .025$	$\alpha = .05$	73	81	90	98	107	117
$\alpha = .01$	$\alpha = .02$	62	69	77	85	93	102
$\alpha = .005$	$\alpha = .01$	55	61	68	76	84	92

1-SIDED	2-SIDED	$n = 29$	$n = 30$	$n = 31$	$n = 32$	$n = 33$	$n = 34$
$\alpha = .05$	$\alpha = .10$	141	152	163	175	188	201
$\alpha = .025$	$\alpha = .05$	127	137	148	159	171	183
$\alpha = .01$	$\alpha = .02$	111	120	130	141	151	162
$\alpha = .005$	$\alpha = .01$	100	109	118	128	138	149

1-SIDED	2-SIDED	$n = 35$	$n = 36$	$n = 37$	$n = 38$	$n = 39$
$\alpha = .05$	$\alpha = .10$	214	228	242	256	271
$\alpha = .025$	$\alpha = .05$	195	208	222	235	250
$\alpha = .01$	$\alpha = .02$	174	186	198	211	224
$\alpha = .005$	$\alpha = .01$	160	171	183	195	208

1-SIDED	2-SIDED	$n = 40$	$n = 41$	$n = 42$	$n = 43$	$n = 44$	$n = 45$
$\alpha = .05$	$\alpha = .10$	287	303	319	336	353	371
$\alpha = .025$	$\alpha = .05$	264	279	295	311	327	344
$\alpha = .01$	$\alpha = .02$	238	252	267	281	297	313
$\alpha = .005$	$\alpha = .01$	221	234	248	262	277	292

1-SIDED	2-SIDED	$n = 46$	$n = 47$	$n = 48$	$n = 49$	$n = 50$
$\alpha = .05$	$\alpha = .10$	389	408	427	446	466
$\alpha = .025$	$\alpha = .05$	361	379	397	415	434
$\alpha = .01$	$\alpha = .02$	329	345	362	380	398
$\alpha = .005$	$\alpha = .01$	307	323	339	356	373

From E. Wilcoxon and R. A. Wilcox, "Some Rapid Approximate Statistical Procedures," 1964. Reprinted by permission of Lederle Labs, a division of the American Cyanamid Co.

TABLE A.15

Factors for Control Charts

	AVERAGES			RANGES	
Number of Items In Sample	Factors for Control Limits		Factors for Central Line	Factors for Control Limits	
n	A_2	A_3	d_2	D_3	D_4
2	1.880	2.659	1.128	0	3.267
3	1.023	1.954	1.693	0	2.575
4	0.729	1.628	2.059	0	2.282
5	0.577	1.427	2.326	0	2.115
6	0.483	1.287	2.534	0	2.004
7	0.419	1.182	2.704	0.076	1.924
8	0.373	1.099	2.847	0.136	1.864
9	0.337	1.032	2.970	0.184	1.816
10	0.308	0.975	3.078	0.223	1.777
11	0.285	0.927	3.173	0.256	1.744
12	0.266	0.886	3.258	0.284	1.716
13	0.249	0.850	3.336	0.308	1.692
14	0.235	0.817	3.407	0.329	1.671
15	0.223	0.789	3.472	0.348	1.652

Adapted from American Society for Testing and Materials, *Manual on Quality Control of Materials*, 1951, Table B2, p. 115. For a more detailed table and explanation, see Acheson J. Duncan, *Quality Control and Industrial Statistics*, 3d ed. Homewood, IL.: Richard D. Irwin, 1974, Table M, p. 927.

APPENDIX B

Answers to Selected Odd-Numbered Quantitative Problems

Chapter 1

1.5. **a.** ratio
b. ratio
c. ordinal
d. nominal
e. ratio
f. ratio
g. nominal
h. ratio

1.7. **a.** 900 electric contractors
b. 35 electric contractors
c. average score for 35 participants
d. average score for all 900 electric contractors

Chapter 2

No answers given

Chapter 3

3.1. 4

3.3. 294

3.5. −1

3.7. 107, 127, 145, 114, 127.5, 143.5

3.9. 6.19, 3.055, 4.96, 7.545, 9.37

3.11. **a.** 8
b. 2.041
c. 6.204
d. 2.491
e. 4
f. 0.69, −0.92, −0.11, 1.89, −1.32, −0.52, 0.29

3.13. **a.** 4.598
b. 4.598

3.15. 58, 631.295, 242.139

3.17. **a.** .75
b. .84
c. .609
d. .902

3.19. **a.** 2.667
b. 11.060
c. 3.326
d. 5
e. −0.85
f. 37.65%

3.21. Between 113 and 137
Between 101 and 149
Between 89 and 161

3.23. 2.236

3.25. 95%, 2.5%, .15%, 16%

3.27. 4.64, 3.59, 1

3.29. 185.694, 13.627

3.31. **a.** 44.9
b. 39
c. 44.82
d. 187.2
e. 13.7

3.33. **a.** 38
b. 25
c. 32.857
d. 251
e. 15.843

3.35. skewed right

3.37. 0.726

3.39. no outliers. negatively skewed

3.41. 2.5, 2, 2, 7, 1, 3, 2

3.43. 38559.6875, 34451, 19757, 36843, 86696, 17729, 44496.5, 77742, 26767.5

3.45. **a.** 392320, 348500
b. 278000, 137920
c. 7387975636, 85953.33406
d. −0.725, +0.877
e. +1.53

3.47. **a.** 33.412, 32.5
b. 58.483, 7.647

3.49. 10.78%, 6.43%

3.51. **a.** 392 to 446, 365 to 473, 338 to 500
b. 79.7%
c. −0.704

3.53. skewed right

3.55. 21.93, 18.14

Chapter 4

4.1. 15, .60

4.3. {4, 8, 10, 14, 16, 18, 20, 22, 26, 28, 30}

4.5. 20, combinations, .60

4.7. 38,760

4.9. **a.** .7167
b. .5000
c. .65
d. .5167

4.11. not solvable

4.13. **a.** .86
b. .31
c. .14

4.15. **a.** .2807
b. .0526
c. .0000
d. .0000

4.17. **a.** .0122
b. .0144

4.19. **a.** .57
b. .3225
c. .4775
d. .5225
e. .6775
f. .0475

4.21. **a.** .039
b. .571
c. .129

4.23. **a.** .2286
b. .2297
c. .3231
d. .0000

4.25. not independent

4.27. **a.** .4054
b. .3261
c. .4074
d. .32

4.29. **a.** .03
b. .2875
c. .3354
d. .9759

4.31. .0538, .5161, .4301

4.33. .7941, .2059

4.35. **a.** .4211
b. .6316
c. .2105
d. .1250
e. .5263
f. .0000
g. .6667
h. .0000

4.37. **a.** .28
b. .04
c. .86
d. .32
e. .1739
f. .66

4.39. **a.** .5410
b. .7857
c. .70
d. .09
e. .2143

4.41. **a.** .39
b. .40
c. .48
d. not independent
e. not mutually exclusive

4.43. **a.** .3483
b. .5317
c. .4683
d. .0817

4.45. **a.** .2625
b. .74375
c. .60
d. .25625
e. .0875

4.47. **a.** .20
b. .6429
c. .40
d. .60
e. .40
f. .3333

4.49. **a.** .469
b. .164
c. .2360
d. .1934
e. .754

4.51. **a.** .2130
b. .4370
c. .2240
d. .6086
e. .3914
f. .8662

4.53. **a.** .276
b. .686
c. .816
d. .59
e. .4023

Chapter 5

5.1. 2.666, 1.8364, 1.3552

5.3. 0.956, 1.1305

5.5. **a.** .0036
b. .1147
c. .3822
d. .5838

5.7. **a.** 14, 2.05
b. 24.5, 3.99
c. 50, 5

5.9. **a.** .0815
b. .0008
c. .227

5.11. **a.** .585
b. .009
c. .013

5.13. **a.** .1032
b. .0000
c. .0352
d. .3480

5.15. **a.** .0538
b. .1539
c. .4142
d. .0672
e. .0244
f. .3702

5.17. **a.** 6.3, 2.51
b. 1.3, 1.14
c. 8.9, 2.98
d. 0.6, .775

5.19. 3.5
a. .0302
b. .1424
c. .0817
d. .42
e. .1009

5.21. **a.** .5488
b. .3293
c. .1220
d. .8913
e. .1912

5.23. **a.** .3012
b. .0000
c. .0336

5.25. **a.** .0104
b. .0000
c. .1653
d. .9636

5.27. **a.** .5091
b. .2937
c. .4167
d. .0014

5.29. **a.** .0529
b. .0294
c. .4235

5.31. **a.** .1333
b. .0238
c. .1143

5.33. .0474

5.35. **a.** .124
b. .849
c. .090
d. .000

5.37. **a.** .1607
b. .7626
c. .3504
d. .5429

5.39. **a.** .1108
b. .017
c. 5
d. .1797
e. .125
f. .0000
g. .056
h. $x = 8(.180)$, $\mu = 8$

5.41. **a.** .2644
b. .0694
c. .0029
d. .7521

5.43. **a.** 5
b. .0244

5.45. **a.** .0687
b. .020
c. .1032
d. 2.28

5.47. .174

5.49. **a.** .3012
b. .1203
c. .7065

5.51. **a.** .0002
b. .0595
c. .2330

5.53. **a.** .0907
b. .0358
c. .1517
d. .8781

5.55. **a.** .265
b. .0136
c. .0067

5.57. **a.** .3854
b. .8333
c. .0981

5.59. **a.** .0539
b. .1603
c. .9315

Chapter 6

6.1. **a.** 1/40
b. 220, 11.547
c. .25
d. .3750
e. .6250

6.3. 2.97, 0.098, .2941

6.5. 981.5, .000294, .2353, .0000, .2353

6.7. **a.** .8944
b. .0122
c. .2144

6.9. **a.** .1788
b. .0329
c. .1476

6.11. **a.** 188.25
b. 244.65
c. 163.81
d. 206.11

6.13. 5.932

6.15. 2.5

6.17. **a.** $P(x \le 16.5 \mid \mu = 21 \text{ and } \sigma = 2.51)$
b. $P(10.5 \le x \le 20.5 \mid \mu = 12.5 \text{ and } \sigma = 2.5)$
c. $P(21.5 \le x \le 22.5 \mid \mu = 24 \text{ and } \sigma = 3.10)$
d. $P(x > 14.5 \mid \mu = 7.2 \text{ and } \sigma = 1.99)$

6.19. **a.** .1170, .120
b. .4090, .415
c. .1985, .196
d. fails test

6.21. .0495

6.23. **a.** .1922
b. .6808
c. .0064
d. .0695

6.27. **a.** .0012
b. .8700
c. .0011
d. .9918

6.29. **a.** .0000
b. .0000
c. .0872
d. .41 minutes

6.31. $\mu = 246.31$
a. .1313
b. .5560

6.33. 15, 15, .1254

6.35. **a.** .1587
b. .0013
c. .6915
d. .9270
e. .0000

6.37. **a.** .0202
b. .9817
c. .1849
d. .4449

6.39. .0000

6.41. **a.** .1131
b. .2912
c. .1543

6.43. .5319, 41.5, .0213

6.45. **a.** .3050
b. .6413
c. .2985
d. .0045

6.47. **a.** .0129
b. .0951
c. .9934
d. .5713

6.49. **a.** .0025
b. .8944
c. .3482

6.51. **a.** .0655
b. .6502
c. .9993

6.53. $11428.57

6.55. **a.** .5488
b. .2592
c. 1.67 months

6.57. 1940, 2018.75, 2269

6.59. .0516, 1.07%

Chapter 7

7.7. 825

7.13. **a.** .0548
b. .7881
c. .0082
d. .8575
e. .1664

7.15. 11.11

7.17. **a.** .9772
b. .2385
c. .1469
d. .1230

7.19. .0000

7.21. **a.** .1894
b. .0559
c. .0000
d. 16.4964

7.23. **a.** .1492
b. .9404
c. .6985
d. .1445
e. .0000

7.25. .26

7.27. **a.** .1977
b. .2843
c. .9881

7.29. **a.** .1020
b. .7568
c. .7019

7.31. 55, 45, 90, 25, 35

7.37. **a.** .3156
b. .00003
c. .1736

7.41. **a.** .0021
b. .9265
c. .0281

7.43. **a.** .0314
b. .2420
c. .2250
d. .1469
e. .0000

7.45. **a.** .8534
b. .0256
c. .0007

7.49. **a.** .6787
b. .0571
c. .0059

7.51. .9147

Chapter 8

8.1. **a.** $24.11 \leq \mu \leq 25.89$
b. $113.17 \leq \mu \leq 126.03$
c. $3.136 \leq \mu \leq 3.702$
d. $54.55 \leq \mu \leq 58.85$

8.3. $45.92 \leq \mu \leq 48.08$

8.5. 66, $62.75 \leq \mu \leq 69.25$

8.7. 5.3, $5.13 \leq \mu \leq 5.47$

8.9. $2.852 \leq \mu \leq 3.760$

8.11. $23.036 \leq \mu \leq 26.030$

8.13. $42.18 \leq \mu \leq 49.06$

8.15. $120.6 \leq \mu \leq 136.2$, 128.4

8.17. $15.631 \leq \mu \leq 16.545$, 16.088

8.19. $2.26886 \leq \mu \leq 2.45346$, 2.36116, .0923

8.21. $36.77 \leq \mu \leq 62.83$

8.23. $7.53 \leq \mu \leq 14.66$

8.25. **a.** $.316 \leq p \leq .704$
b. $.777 \leq p \leq .863$
c. $.456 \leq p \leq .504$
d. $.246 \leq p \leq .394$

8.27. $.38 \leq p \leq .56$
$.36 \leq p \leq .58$
$.33 \leq p \leq .61$

8.29. **a.** $.4287 \leq p \leq .5113$
b. $.2488 \leq p \leq .3112$

8.31. **a.** .266
b. $.247 \leq p \leq .285$

8.33. $.5935 \leq p \leq .6665$

8.35. **a.** $18.46 \leq \sigma^2 \leq 189.73$
b. $0.64 \leq \sigma^2 \leq 7.46$
c. $645.45 \leq \sigma^2 \leq 1923.10$
d. $12.61 \leq \sigma^2 \leq 31.89$

8.37. $9.71 \leq \sigma^2 \leq 46.03$, 18.49

8.39. $14{,}084{,}038.51 \leq \sigma^2 \leq 69{,}553{,}848.45$

8.41. **a.** 2522
b. 601
c. 268
d. 16,577

8.43. 106

8.45. 1,083

8.47. 97

8.49. 12.03, $11.78 \leq \mu \leq 12.28$, $11.72 \leq \mu \leq 12.34$, $11.58 \leq \mu \leq 12.48$

8.51. $29.133 \leq \sigma^2 \leq 148.235$, $25.911 \leq \sigma^2 \leq 182.529$

8.53. $9.19 \leq \mu \leq 12.34$

8.55. $2.307 \leq \sigma^2 \leq 15.374$

8.57. $36.231 \leq \mu \leq 38.281$

8.59. $.542 \le p \le .596$, .569

8.61. $5.892 \le \mu \le 7.542$

8.63. $.726 \le p \le .814$

8.65. $34.11 \le \mu \le 53.29$, $101.44 \le \sigma^2 \le 821.35$

8.67. $-0.20 \le \mu \le 5.16$, 2.48

8.69. 543

8.71. $.0026 \le \sigma^2 \le .0071$

Chapter 9

9.1. **a.** $z = 2.77$, reject
b. .0028
c. 22.115, 27.885

9.3. **a.** $z = 1.59$, reject
b. .0559
c. 1212.04

9.5. $z = 1.84$, fail to reject

9.7. $z = 1.46$, fail to reject

9.9. $z = 2.99$, .0014, reject

9.11. $t = 0.56$, fail to reject

9.13. $t = 2.44$, reject

9.15. $t = 1.59$, fail to reject

9.17. $t = -3.31$, reject

9.19. $t = -2.02$, fail to reject

9.21. fail to reject

9.23. $z = -1.66$, fail to reject

9.25. $z = -1.89$, fail to reject

9.27. $z = 1.22$, fail to reject,
$z = 1.34$, fail to reject

9.29. $z = -3.11$, reject

9.31. **a.** $\chi^2 = 22.4$, fail to reject
b. $\chi^2 = 42$, reject
c. $\chi^2 = 2.64$, fail to reject
d. $\chi^2 = 2.4$, reject

9.33. $\chi^2 = 21.7$, fail to reject

9.35. $\chi^2 = 17.34$, reject

9.37. **a.** $\beta = .8159$
b. $\beta = .7422$
c. $\beta = .5636$
d. $\beta = .3669$

9.39. **a.** $\beta = .3632$
b. $\beta = .0122$
c. $\beta = .0000$

9.41. $z = -0.48$, fail to reject, .6293, .1492, .0000

9.43. $t = -1.98$, reject

9.45. $\chi^2 = 32.675$, fail to reject

9.47. $z = -1.34$, fail to reject

9.49. $z = -3.72$, reject

9.51. $t = -5.70$, reject

9.53. $\chi^2 = 106.47$, reject

9.55. $t = -2.80$, reject

9.57. $z = 3.96$, reject

9.59. $t = 4.50$, reject

9.61. $\chi^2 = 45.866$, reject

Chapter 10

10.1. **a.** $z = -1.01$, fail to reject
b. -2.41
c. .1562

10.3. **a.** $z = 5.48$, reject
b. $4.04 \le \mu_1 - \mu_2 \le 10.02$

10.5. $-1.86 \le \mu_1 - \mu_2 \le -0.54$

10.7. $z = -2.32$, fail to reject

10.9. $z = -2.27$, reject

10.11. $t = -1.05$, fail to reject

10.13. $t = -4.64$, reject

10.15. **a.** $1905.38 \le \mu_1 - \mu_2 \le 3894.62$
b. $t = -4.91$, reject

10.17. $t = 2.06$, reject

10.19. $t = 4.95$, reject, $2258.05 \le \mu_1 - \mu_2 \le 5541.95$

10.21. $t = 3.31$, reject

10.23. $26.29 \le D \le 54.83$

10.25. $-3415.6 \le D \le 6021.2$

10.27. $6.58 \le D \le 49.60$

10.29. $63.71 \le D \le 86.29$

10.31. **a.** $z = 0.75$, fail to reject
b. $z = 4.83$, reject

10.33. $z = -3.35$, reject

10.35. $z = -0.94$, fail to reject

10.37. $z = 2.35$, reject

10.39. $F = 1.80$, fail to reject

10.41. $F = 0.81$, fail to reject

10.43. $F = 1.53$, fail to reject

10.45. $z = -2.38$, reject

10.47. $t = 0.85$, fail to reject

10.49. $t = -5.26$, reject

10.51. $z = -1.20$, fail to reject

10.53. $F = 1.24$, fail to reject

10.55. $-3.201 \le D \le 2.313$

10.57. $F = 1.31$, fail to reject

10.59. $t = 2.97$, reject

10.61. $z = 6.78$, reject

10.63. $3.553 \le D \le 5.447$

10.65. $t = 6.71$, reject

10.67. $.142 \le p_1 - p_2 \le .250$

10.69. $z = 8.86$, reject

10.71. $t = 4.52$, reject

Chapter 11

11.5. $F = 11.07$, reject

11.7. $F = 13.00$, reject

11.9. 4, 50, 54, 145.8975, 19.4436, $F = 7.50$, reject

11.11. $F = 10.10$, reject

11.13. $F = 11.76$, reject

11.15. 4 levels; sizes 18, 15, 21, and 11; $F = 2.95$, $p = .04$; means = 226.73, 238.79, 232.58, and 239.82.

11.17. HSD = 0.896, groups 3 & 6 significantly different

11.19. HSD = 1.586, groups 1 & 2 significantly different

11.21. HSD = 10.27, groups 1 & 3 significantly different

11.23. $HSD_{1,3} = .0381$, groups 1 & 3 significantly different

11.25. $HSD_{1,3} = 1.764$, $HSD_{2,3} = 1.620$, groups 1 & 3 and 2 & 3 significantly different

11.29. $F = 1.48$, fail to reject

11.31. $F = 3.90$, fail to reject

11.33. $F = 15.37$, reject

11.37. 2, 1, 4 row levels, 3 column levels, yes
$df_{row} = 3$, $df_{col.} = 2$, $df_{int.} = 6$, $df_{error} = 12$, $df_{total} = 23$

11.39. $MS_{row} = 1.047$, $MS_{col.} = 1.281$, $MS_{int.} = 0.258$, $MS_{error} = 0.436$, $F_{row} = 2.40$, $F_{col.} = 2.94$, $F_{int.} = 0.59$,
fail to reject any hypothesis

11.41. $F_{row} = 87.25$, reject; $F_{col.} = 63.67$, reject; $F_{int.} = 2.07$, fail to reject

11.43. $F_{row} = 34.31$, reject; $F_{col.} = 14.20$, reject; $F_{int.} = 3.32$, reject

11.45. no significant interaction or row effects; significant column effects.

11.47. $F = 8.82$, reject; HSD = 3.33 groups 1 & 2, 2 & 3, and 2 & 4 significantly different.

11.49. $df_{treat.} = 5$, $MS_{treat.} = 42.0$, $df_{error} = 36$, $MS_{error} = 18.194$, $F = 2.31$

11.51. 1 treatment variable, 3 levels; 1 blocking variable, 6 levels; $df_{treat.} = 2$, $df_{block} = 5$, $df_{error} = 10$

11.53. $F_{treat.} = 31.51$, reject; $F_{blocks} = 43.20$, reject; HSD = 8.757, no pairs significant

11.55. $F_{rows} = 38.21$, reject; $F_{col.} = 0.23$, fail to reject; $F_{inter} = 1.30$, fail to reject

11.57. $F = 7.38$, reject

11.59. $F = 0.46$, fail to reject

11.61. $F_{treat.} = 13.64$, reject

Chapter 12

12.1. −0.927

12.3. 0.645

12.5. 0.975, 0.985, 0.957

12.7. $\hat{y} = 144.414 - 0.898x$

12.9. $\hat{y} = 15.460 - 0.715x$

12.11. $\hat{y} = 600.186 - 72.328x$

12.13. $\hat{y} = 13.625 + 2.303x$, −1.1694, 3.9511, −1.3811, 2.7394, −4.1401

12.15. 18.6597, 37.5229, 51.8948, 62.6737, 86.0281, 118.3648, 122.8561; 6.3403, −8.5229, −5.8948, 7.3263, 1.9720, −6.3648, 5.1439

12.17. 4.0259, 11.1722, 9.7429, 12.6014, 10.4576; 0.9741, 0.8278, −0.7429, 2.3986, −3.4575

12.19. 4.7244, −0.9836, −0.3996, −6.7537, 2.7683, 0.6442; No apparent violations

12.21. The error terms appear to be non independent

12.23. Violation of the homoscedasticity assumption

12.25. SSE = 272.0, $s_e = 7.376$, 6 out of 7 and 7 out of 7

12.27. SSE = 19.8885, $s_e = 2.575$

12.29. $s_e = 4.391$

12.31. $\hat{y} = 118.257 - 0.1504x$, $s_e = 40.526$

12.33. $r^2 = .972$

12.35. $r^2 = .685$

12.37. $\hat{y} = -599.3674 + 19.2204x$, $s_e = 13.539$, $r^2 = .688$

12.39. $t = -13.18$, reject

12.41. $t = -2.56$, fail to reject

12.43. F is significant at $\alpha = .05$, $t = 2.874$, reject at $\alpha = .05$

12.45. $38.523 \le y \le 70.705$, $10.447 \le y \le 44.901$

12.47. $0.97 \le E(y_{10}) \le 15.65$

12.49. $\hat{y} = 1366461.25 - 678.9643x$, $\hat{y}(2010) = 1743.04$

12.51. $r = -.94$

12.53. **a.** $\hat{y} = -11.335 + 0.355x$
b. 7.48, 5.35, 3.22, 6.415, 9.225, 10.675, 4.64, 9.965, −2.48, −0.35, 3.78, −2.415, 0.745, 1.325, −1.64, 1.035
c. SSE = 32.4649
d. $s_e = 2.3261$
e. $r^2 = .608$
f. $t = 3.05$, reject

12.55. **a.** $20.92 \le E(y_{60}) \le 26.8$
b. $20.994 \le y \le 37.688$

12.57. $r^2 = .826$

12.59. $\hat{y} = -0.863565 + 0.92025x$; $r^2 = .405$

12.61. $r = .8998$

12.63. $\hat{y} = -39.0071 + 66.36277x$, $r^2 = .906$, $s_e = 21.13$

12.65. $\hat{y} = 3670.082 - 6.62083x$, $s_e = 1337.556$, $r^2 = .24$, $t = -1.26$, fail to reject

Chapter 13

13.1. $\hat{y} = 25.03 - 0.0497x_1 + 1.928x_2$, 28.586

13.3. $\hat{y} = 121.62 - 0.174x_1 + 6.02x_2 + 0.00026x_3 + 0.0041x_4$, 4

13.5. Per capita consumption = −7,629.627 + 116.2549 paper consumption − 120.0904 fish consumption + 45.73328 gasoline consumption

13.7. 9, fail to reject null overall at $\alpha = .05$, only $t = 2.73$ for x_1, significant at $\alpha = .05$, $s_e = 3.503$, $R^2 = .408$, adj. $R^2 = .203$

13.9. Per capita consumption = − 7,629.627 + 116.2549 paper consumption − 120.0904 fish consumption + 45.73328 gasoline consumption; $F = 14.319$ with p-value = .0023; $t = 2.67$ with p-value = .032 for gasoline consumption. The p-values of the t statistics for the other two predictors are insignificant.

13.11. $\hat{y} = 3.981 + 0.07322x_1 - 0.03232x_2 - 0.003886x_3$, $F = 100.47$ significant at $\alpha = .001$, $t = 3.50$ for x_1 significant at $\alpha = .01$, $s_e = 0.2331$, $R^2 = .965$, adj. $R^2 = .955$

13.13. 3 predictors, 15 observations, $\hat{y} = 657.053 + 5.710\,x_1 - 0.417\,x_2 - 3.471\,x_3$, $R^2 = .842$, adjusted $R^2 = .630$, $s_e = 109.43$, $F = 8.96$ with $p = .0027$, x_1 significant at $\alpha = .01$, x_3 significant at $\alpha = .05$

13.15. $s_e = 9.722$, $R^2 = .515$, adjusted $R^2 = .404$

13.17. $s_e = 6.544$, $R^2 = .005$, adjusted $R^2 = .000$

13.19. model with x_1, x_2: $s_e = 6.333$, $R^2 = .963$, adjusted $R^2 = .957$ model with x_1: $s_e = 6.124$, $R^2 = .963$, adjusted $R^2 = .960$

13.21. heterogeneity of variance

13.23. 2, $\hat{y} = 203.3937 + 1.1151x_1 - 2.2115x_2$, $F = 24.55$, reject, $R^2 = .663$, adjusted $R^2 = .636$

13.25. $\hat{y} = 362 - 4.75x_1 - 13.9x_2 + 1.87x_3$; $F = 16.05$, reject; $s_e = 37.07$; $R^2 = .858$; adjusted $R^2 = .804$; x_1 only significant predictor

13.27. Employment = 71.03 + 0.4620 Naval Vessels + 0.02082 Commercial

$F = 1.22$, fail to reject; $R^2 = .379$; adjusted $R^2 = .068$; no significant predictors

13.29. Corn = −2718 + 6.26 Soybeans − 0.77 Wheat; $F = 14.25$, reject; $s_e = 862.4$; $R^2 = .803$; adjusted $R^2 = .746$; Soybeans was a significant predictor

Chapter 14

14.1. Simple Model: $\hat{y} = -147.27 + 27.128x$, $F = 229.67$ with $p = .000$, $s_e = 27.27$, $R^2 = .97$, adjusted $R^2 = .966$ Quadratic Model: $\hat{y} = -22.01 + 3.385x_1 + 0.9373x_2$, $F = 578.76$ with $p = .000$, $s_e = 12.3$, $R^2 = .995$, adjusted $R^2 = .993$, for $x_1 : t = 0.75$, for $x_2 : t = 5.33$

14.3. $\hat{y} = 1012 - 14.1x + 0.611x^2$; $R^2 = .947$; $s_e = 605.7$; adjusted $R^2 = .911$; $t(x) = -0.17$, fail to reject; $t(x^2) = 1.03$, fail to reject

14.5. $\hat{y} = -28.61 - 2.68x_1 + 18.25x_2 - 0.2135x_1^2 - 1.533x_2^2 + 1.226x_1x_2$; $F = 63.43$, reject; $s_e = 4.669$, $R^2 = .958$; adjusted $R^2 = .943$; no significant t ratios. Model with no interaction term: $R^2 = .957$

14.7. $\hat{y} = 13.619 - 0.01201x_1 + 2.988x_2$, $F = 8.43$ significant at $\alpha = .01$, $t = 3.88$ for x_2, (dummy variable) significant at $\alpha = .01$, $s_e = 1.245$, $R^2 = .652$, adj. $R^2 = .575$

14.9. x_1 and x_2 are significant predictors at $\alpha = .05$

14.11. Price = 7.066 − 0.0855 Hours + 9.614 Probability + 10.507 French Quarter, $F = 6.80$ significant at $\alpha = .01$, $t = 3.97$ for French Quarter (dummy variable) significant at $\alpha = .01$, $s_e = 4.02$, $R^2 = .671$, adj. $R^2 = .573$

14.13. Step 1: x_2 entered, $t = -7.53$, $r^2 = .794$
Step 2: x_3 entered, $t_2 = -4.60$, $t_3 = 2.93$, $R^2 = .876$

14.15. 4 predictors, x_4 and x_5 are not in model.

14.17. Step 1: Dividends in the model, $t = 6.69$, $r^2 = .833$
Step 2: Net income and dividends in model, $t = 2.24$ and $t = 4.36$, $R^2 = .897$

14.19.

	y	x_1	x_2
x_1	−.653		
x_2	−.891	.650	
x_3	.821	−.615	−.688

14.21.

	Net Income	Dividends
Dividends	.682	
Underwriting	.092	−.522

14.23. $\hat{y} = 564 - 27.99\,x_1 - 6.155\,x_2 - 15.90\,x_3$, $R^2 = .809$, adjusted $R^2 = .738$, $s_e = 42.88$, $F = 11.32$ with $p = .003$, x_2 only significant predictor x_1 is a non-significant indicator variable

14.25. The procedure stopped at step 1 with only log x in the model, = −13.20 + 11.64 log x_1, $R^2 = .9617$

14.27. The procedure went 2 steps, step 1: silver entered, $R^2 = .5244$, step 2: aluminum entered, $R^2 = .8204$, final model: gold = −50.19 + 18.9 silver + 3.59 aluminum

14.29. The procedure went 3 steps, step 1: food entered, $R^2 = .84$, step 2: fuel oil entered, $R^2 = .95$, step 3: shelter entered, $R^2 = .96$, final model: All = −1.0615 + 0.474 food + 0.269 fuel oil + 0.249 shelter

14.31. Grocery = 76.23 + 0.08592 Housing + 0.16767 Utility + 0.0284 Transportation − 0.0659 Healthcare, $F = 2.29$ not significant; $s_e = 4.416$; $R^2 = .315$; Adjusted $R^2 = .177$; Utility only significant predictor.

Chapter 15

15.1. MAD = 1.367, MSE = 2.27

15.3. MAD = 3.583, MSE = 15.765

15.5. **a.** 44.75, 52.75, 61.50, 64.75, 70.50, 81
b. 53.25, 56.375, 62.875, 67.25, 76.375, 89.125

15.7. $\alpha = .3$: 9.4, 9, 8.7, 8.8, 9.1, 9.7, 9.9, 9.8
$\alpha = .7$: 9.4, 8.6, 8.1, 8.7, 9.5, 10.6, 10.4, 9.8

15.9. $\alpha = .2$: 332, 404.4, 427.1, 386.1, 350.7, 315, 325.2, 362.6, 423.5, 453, 477.4, 554.9

$\alpha = .9$: 332, 657.8, 532, 253, 213.4, 176.1, 347, 495.5, 649.9, 578.9, 575.4, 836; $\text{MAD}_{\alpha=.2} = 190.8$; $\text{MAD}_{\alpha=.9} = 168.6$

15.11. Members = 145392.3 − 64.6354 year, $R^2 = 91.44\%$, $s_e = 215.1158$, $F = 117.4$ reject

15.13. TC: 136.78, 132.90, 128.54, 126.43, 124.86, 122, 119.08, 116.76, 114.61, 112.70, 111.75, 111.36

SI: 93.30, 90.47, 92.67, 98.77, 111.09, 100.83, 113.52, 117.58, 112.36, 92.08, 99.69, 102.73

15.15. $D = 1.276$, reject the null hypothesis—significant autocorrelation

15.17. $D = 2.49$, no significant autocorrelation

15.19. 1 lag: Housing Starts = −8.87 + 1.06 lag 1; R^2 = 89.2%; s_e = 48.52
2 lags: Housing Starts = 13.66 + 1.0569 lag 2; R^2 = 75.9%; s_e = 70.84

15.21. **a.** 100, 139.9, 144, 162.6, 200, 272.8, 310.7, 327.1, 356.6, 376.9, 388.8, 398.9
b. 32.2, 45, 46.4, 52.3, 64.4, 87.8, 100, 105.3, 114.8, 121.3, 125.1, 128.4

15.23. 100, 103.2, 124.8

15.25. 121.6, 127.4, 131.4

15.27. **a.** Linear: = 9.96 − 0.14 x, R^2 = 90.9%, Quadratic: = 10.4 − 0.252 x + .00445 x_2, R^2 = 94.4%
b. MAD = .3385
c. MAD (α = .3) = .4374, MAD (α = .7) = .2596
d. α = .7 did best
e. 100.28, 101.51, 99.09, 99.12

15.29. 100, 104.8, 114.5, 115.5, 114.1

15.31. $MAD_{mov.avg.}$ = 540.44, $MAD_{\alpha=.2}$ = 846.43

15.33. Jan. 95.35, Feb. 99.69, March 106.75, April 103.99, May 100.99, June 106.96, July 94.53, Aug. 99.60, Sept. 104.16, Oct. 97.04, Nov. 95.75, Dec. 95.19

15.35. Laspeyres: 105.2, 111.0; Paasche: 105.1, 110.8

15.37. MSE_{ma} = 123.4; MSE_{wma} = 79.39

15.39. 98.07, 103.84, 97.04, 101.05

15.43. D = 0.84, reject

15.45. D = 0.98, reject

Chapter 16

16.1. χ^2 = 18.095, reject.

16.3. χ^2 = 2.001, fail to reject, λ = 0.9.

16.5. χ^2 = 198.48, reject.

16.7. χ^2 = 2.45, fail to reject

16.9. χ^2 = 3.398, fail to reject

16.11. χ^2 = 0.00, fail to reject

16.13. χ^2 = 34.97, reject

16.15. χ^2 = 6.43, reject

16.17. χ^2 = 3.93, fail to reject

16.19. χ^2 = 1.652, fail to reject

16.21. χ^2 = 14.91, reject

16.23. χ^2 = 7.25, fail to reject

16.25. χ^2 = 59.63, reject

16.27. χ^2 = 54.63, reject

Chapter 17

17.1. R = 11, fail to reject

17.3. $\alpha/2$ = .025, p-value = .0264, fail to reject

17.5. R = 27, z = −1.08, fail to reject

17.7. U = 26.5, p-value = .6454, fail to reject

17.9. U = 11, p-value = .0156, fail to reject

17.11. z = −3.78, reject

17.13. z = −2.59, reject

17.15. z = −3.20, reject

17.17. z = −1.75, reject

17.19. K = 21.21, reject

17.21. K = 2.75, fail to reject

17.23. K = 18.99, reject

17.25. χ^2 = 13.8, reject

17.27. χ^2 = 14.8, reject

17.29. 4, 5, S = 2.04, fail to reject

17.31. r_s = .893

17.33. r_s = −.95

17.35. r_s = −.398

17.37. r_s = −.855

17.39. U = 20, p-value = .2344, fail to reject

17.41. K = 7.75, fail to reject

17.43. r_s = −.81

17.45. z = −0.40, fail to reject

17.47. z = 0.96, fail to reject

17.49. U = 45.5, p-value = .739, fail to reject

17.51. z = −1.91, fail to reject

17.53. R = 21, fail to reject

17.55. z = −2.43, reject

17.57. K = 17.21, reject

17.59. K = 11.96, reject

Chapter 18

18.5. $\bar{\bar{x}}$ = 4.51, UCL = 5.17, LCL = 3.85
$\bar{R}$ = 0.90, UCL = 2.05, LCL = 0

18.7. p = .05, UCL = .1534, LCL = .000

18.9. $\bar{c}$ = 1.34375, UCL = 4.82136, LCL = .000

18.11. Chart 1: nine consecutive points below centerline, four out of five points in the outer 2/3 of the lower region
Chart 2: eight consecutive points above the centerline
Chart 3: in control

18.15. p = .104, LCL = 0.000, UCL = .234

18.17. $\bar{c}$ = 2.13889, UCL = 6.52637, LCL = .0000

18.19. $\bar{\bar{x}}$ = 14.99854, UCL = 15.02269, LCL = 14.97439
$\bar{R}$ = .05, UCL = .1002, LCL = .0000

18.21. $\bar{c}$ = 0.64, UCL = 3.04, LCL = .0000

18.23. p = 0.06, LCL = 0.000, UCL = .1726

Chapter 19 (On Wiley Web site)

19.1. **a.** 390
b. 70
c. 82, 296
d. 140

19.3. 60, 10

19.7. 31.75, 6.50

19.9. Lock in = 85, 182.5, 97.5

19.11. **a.** 75,000
b. Avoider
c. >75,000

19.13. 244.275, 194.275

19.15. 21012.32, 12.32

19.17. **b.** 267.5, 235
c. 352.5, 85

19.19. **a.** 2000, 200
b. 500

19.21. 875,650

19.23. Reduction: .60, .2333, .1667
Constant: .10, .6222, .2778
Increase: .0375, .0875, .8750, 21425.55, 2675.55

GLOSSARY

A

a posteriori After the experiment; pairwise comparisons made by the researcher *after* determining that there is a significant overall F value from ANOVA; also called *post hoc*.

a priori Determined before, or prior to, an experiment.

adjusted R^2 A modified value of R^2 in which the degrees of freedom are taken into account, thereby allowing the researcher to determine whether the value of R^2 is inflated for a particular multiple regression model.

after-process quality control A type of quality control in which product attributes are measured by inspection after the manufacturing process is completed to determine whether the product is acceptable.

all possible regressions A multiple regression search procedure in which all possible multiple linear regression models are determined from the data using all variables.

alpha (α) The probability of committing a Type I error; also called the level of significance.

alternative hypothesis The hypothesis that complements the null hypothesis; usually it is the hypothesis that the researcher is interested in proving.

analysis of variance (ANOVA) A technique for statistically analyzing the data from a completely randomized design; uses the F test to determine whether there is a significant difference in two or more independent groups.

arithmetic mean The average of a group of numbers.

autocorrelation A problem that arises in regression analysis when the data occur over time and the error terms are correlated; also called serial correlation.

autoregression A multiple regression forecasting technique in which the independent variables are time-lagged versions of the dependent variable.

averaging models Forecasting models in which the forecast is the average of several preceding time periods.

B

backward elimination A step-by-step multiple regression search procedure that begins with a full model containing all predictors. A search is made to determine if there are any nonsignificant independent variables in the model. If there are no nonsignificant predictors, then the backward process ends with the full model. If there are nonsignificant predictors, then the predictor with the smallest absolute value of t is eliminated and a new model is developed with the remaining variables. This procedure continues until only variables with significant t values remain in the model.

bar graph A bar graph is a chart that contains two or more categories along one axis and a series of bars, one for each category, along the other axis. Usually the length of the bar represents the magnitude of the measure for each category. A bar graph is qualitative and may be either horizontal or vertical.

Bayes' rule An extension of the conditional law of probabilities discovered by Thomas Bayes that can be used to revise probabilities.

benchmarking A quality control method in which a company attempts to develop and establish total quality management from product to process by examining and emulating the best practices and techniques used in their industry.

beta (β) The probability of committing a Type II error.

bimodal Data sets that have two modes.

binomial distribution Widely known discrete distribution in which there are only two possibilities on any one trial.

blocking variable A variable that the researcher wants to control but is not the treatment variable of interest.

bounds The error portion of the confidence interval that is added and/or subtracted from the point estimate to form the confidence interval.

box-and-whisker plot A diagram that utilizes the upper and lower quartiles along with the median and the two most extreme values to depict a distribution graphically; sometimes called a box plot.

C

***c* chart** A quality control chart for attribute compliance that displays the number of nonconformances per item or unit.

categorical data Non numerical data that are frequency counts of categories from one or more variables.

cause-and-effect diagram A tool for displaying possible causes for a quality problem and the interrelationships among the causes; also called a fishbone diagram or an Ishikawa diagram.

census A process of gathering data from the whole population for a given measurement of interest.

centerline The middle horizontal line of a control chart, often determined either by a product or service specification or by computing an expected value from sample information.

central limit theorem A theorem that states that regardless of the shape of a population, the distributions of sample means and proportions are normal if sample sizes are large.

Chebyshev's theorem A theorem stating that at least $1 - 1/k^2$ values will fall within $\pm k$ standard deviations of the mean regardless of the shape of the distribution.

check sheet Simple forms consisting of multiple categories and columns for recording tallies for displaying the frequency of outcomes for some quality-related event or activity.

chi-square distribution A continuous distribution determined by the sum of the squares of k independent random variables.

chi-square goodness-of-fit test A statistical test used to analyze probabilities of multinomial distribution trials along a single dimension; compares expected, or theoretical, frequencies of categories from a population distribution to the observed, or actual, frequencies from a distribution.

chi-square test of independence A statistical test used to analyze the frequencies of two variables with multiple categories to determine whether the two variables are independent.

class mark Another name for class midpoint; the midpoint of each class interval in grouped data.

class midpoint For any given class interval of a frequency distribution, the value halfway across the class interval; the average of the two class endpoints.

classical method of assigning probabilities Probabilities assigned based on rules and laws.

classification variable The independent variable of an experimental design that was present prior to the experiment and is not the result of the researcher's manipulations or control.

classifications The subcategories of the independent variable used by the researcher in the experimental design; also called levels.

cluster (or area) sampling A type of random sampling in which the population is divided into nonoverlapping areas or clusters and elements are randomly sampled from the areas or clusters.

coefficient of correlation (r) A statistic developed by Karl Pearson to measure the linear correlation of two variables.

coefficient of determination (r^2) The proportion of variability of the dependent variable accounted for or explained by the independent variable in a regression model.

coefficient of multiple determination (R^2) The proportion of variation of the dependent variable accounted for by the independent variables in the regression model.

coefficient of skewness A measure of the degree of skewness that exists in a distribution of numbers; compares the mean and the median in light of the magnitude of the standard deviation.

coefficient of variation (CV) The ratio of the standard deviation to the mean, expressed as a percentage.

collectively exhaustive events A list containing all possible elementary events for an experiment.

combinations Used to determine the number of possible ways n things can happen from N total possibilities when sampling without replacement.

complement of a union The only possible case other than the union of sets X and Y; the probability that neither X nor Y is in the outcome.

complementary events Two events, one of which comprises all the elementary events of an experiment that are not in the other event.

completely randomized design An experimental design wherein there is one treatment or independent variable with two or more treatment levels and one dependent variable. This design is analyzed by analysis of variance.

concomitant variables Variables that are not being controlled by the researcher in the experiment but can have an effect on the outcome of the treatment being studied; also called confounding variables.

conditional probability The probability of the occurrence of one event given that another event has occurred.

confidence interval A range of values within which the analyst can declare, with some confidence, the population parameter lies.

confounding variables Variables that are not being controlled by the researcher in the experiment but can have an effect on the outcome of the treatment being studied; also called concomitant variables.

contingency analysis Another name for the chi-square test of independence.

contingency table A two-way table that contains the frequencies of responses to two questions; also called a raw values matrix.

continuous distributions Distributions constructed from continuous random variables.

continuous random variables Variables that take on values at every point over a given interval.

control chart A quality control graph that contains an upper control limit, a lower control limit, and a centerline; used to evaluate whether a process is or is not in a state of statistical control.

convenience sampling A nonrandom sampling technique in which items for the sample are selected for the convenience of the researcher.

correction for continuity A correction made when a binomial distribution problem is approximated by the normal distribution because a discrete distribution problem is being approximated by a continuous distribution.

correlation A measure of the degree of relatedness of two or more variables.

covariance The variance of x and y together.

critical value The value that divides the nonrejection region from the rejection region.

critical value method A method of testing hypotheses in which the sample statistic is compared to a critical value in order to reach a conclusion about rejecting or failing to reject the null hypothesis.

cumulative frequency A running total of frequencies through the classes of a frequency distribution.

cycles Patterns of highs and lows through which data move over time periods usually of more than a year.

cyclical effects The rise and fall of time-series data over periods longer than 1 year.

D

decision alternatives The various choices or options available to the decision maker in any given problem situation.

decision analysis A category of quantitative business techniques particularly targeted at clarifying and enhancing the decision-making process.

decision making under certainty A decision-making situation in which the states of nature are known.

decision making under risk A decision-making situation in which it is uncertain which states of nature will occur but the probability of each state of nature occurring has been determined.

decision making under uncertainty A decision-making situation in which the states of nature that may occur are unknown and the probability of a state of nature occurring is also unknown.

decision table A matrix that displays the decision alternatives, the states of nature, and the payoffs for a particular decision-making problem; also called a payoff table.

decision trees A flowchart-like depiction of the decision process that includes the various decision alternatives, the various states of nature, and the payoffs.

decomposition Breaking down the effects of time-series data into the four component parts of trend, cyclical, seasonal, and irregular.

degrees of freedom A mathematical adjustment made to the size of the sample; used along with α to locate values in statistical tables.

dependent samples Two or more samples selected in such a way as to be dependent or related; each item or person in one sample has a corresponding matched or related item in the other samples. Also called related samples.

dependent variable In regression analysis, the variable that is being predicted.

descriptive statistics Statistics that have been gathered on a group to describe or reach conclusions about that same group.

deseasonalized data Time-series data in which the effects of seasonality have been removed.

Design for Six Sigma A quality scheme, an offshoot of Six Sigma, that places an emphasis on designing a product or process right the first time thereby allowing organizations the opportunity to reach even higher sigma levels through Six Sigma.

deterministic model Mathematical models that produce an "exact" output for a given input.

deviation from the mean The difference between a number and the average of the set of numbers of which the number is a part.

discrete distributions Distributions constructed from discrete random variables.

discrete random variables Random variables in which the set of all possible values is at most a finite or a countably infinite number of possible values.

disproportionate stratified random sampling A type of stratified random sampling in which the proportions of items selected from the strata for the final sample do not reflect the proportions of the strata in the population.

dot plot A dot plot is a relatively simple statistical chart used to display continuous quantitative data where each data value is plotted along the horizontal axis and is represented on the chart by a dot.

dummy variable Another name for a qualitative or indicator variable; usually coded as 0 or 1 and represents whether or not a given item or person possesses a certain characteristic.

Durbin-Watson test A statistical test for determining whether significant autocorrelation is present in a time-series regression model.

E

elementary events Events that cannot be decomposed or broken down into other events.

empirical rule A guideline that states the approximate percentage of values that fall within a given number of standard deviations of a mean of a set of data that are normally distributed.

EMV'er A decision maker who bases his or her decision on the expected monetary value of the decision alternative.

error of an individual forecast The difference between the actual value and the forecast of that value.

error of estimation The difference between the statistic computed to estimate a parameter and the parameter.

event An outcome of an experiment.

expected monetary value (EMV) A value of a decision alternative computed by multiplying the probability of each state of nature by the state's associated payoff and summing these products across the states of nature.

expected value The long-run average of occurrences; sometimes referred to as the mean value.

expected value of perfect information The difference between the payoff that would occur if the decision maker knew which states of nature would occur and the expected monetary payoff from the best decision alternative when there is no information about the occurrence of the states of nature.

expected value of sample information The difference between the expected monetary value with information and the expected monetary value without information.

experiment A process that produces outcomes.

experimental design A plan and a structure to test hypotheses in which the researcher either controls or manipulates one or more variables.

exponential distribution A continuous distribution closely related to the Poisson distribution that describes the times between random occurrences.

exponential smoothing A forecasting technique in which a weighting system is used to determine the importance of previous time periods in the forecast.

F

***F* distribution** A distribution based on the ratio of two random variances; used in testing two variances and in analysis of variance.

***F* value** The ratio of two sample variances, used to reach statistical conclusions regarding the null hypothesis; in ANOVA, the ratio of the treatment variance to the error variance.

factorial design An experimental design in which two or more independent variables are studied simultaneously and every level of each treatment is studied under the conditions of every level of all other treatments. Also called a factorial experiment.

factors Another name for the independent variables of an experimental design.

Failure Mode and Effects Analysis (FMEA) A systematic way for identifying the effects of potential product or process failure. It includes methodology for eliminating or reducing the chance of a failure occurring.

finite correction factor A statistical adjustment made to the z formula for sample means; adjusts for the fact that a population is finite and the size is known.

first-differences approach A method of transforming data in an attempt to reduce or remove autocorrelation from a time-series regression model; results in each data value being subtracted from each succeeding time period data value, producing a new, transformed value.

fishbone diagram A display of possible causes of a quality problem and the interrelationships among the causes. The problem is diagrammed along the main line of the "fish" and possible causes are diagrammed as line segments angled off in such a way as to give the appearance of a fish skeleton. Also called an Ishikawa diagram or a cause-and-effect diagram.

flowchart A schematic representation of all the activities and interactions that occur in a process.

forecasting The art or science of predicting the future.

forecasting error A single measure of the overall error of a forecast for an entire set of data.

forward selection A multiple regression search procedure that is essentially the same as stepwise regression analysis except that once a variable is entered into the process, it is never deleted.

frame A list, map, directory, or some other source that is being used to represent the population in the process of sampling.

frequency distribution A summary of data presented in the form of class intervals and frequencies.

frequency polygon A graph constructed by plotting a dot for the frequencies at the class midpoints and connecting the dots.

Friedman test A nonparametric alternative to the randomized block design.

G

grouped data Data that have been organized into a frequency distribution.

H

heteroscedasticity The condition that occurs when the error variances produced by a regression model are not constant.

histogram A type of vertical bar chart constructed by graphing line segments for the frequencies of classes across the class intervals and connecting each to the x axis to form a series of rectangles.

homoscedasticity The condition that occurs when the error variances produced by a regression model are constant.

Hurwicz criterion An approach to decision making in which the maximum and minimum payoffs selected from each decision alternative are used with a weight, α, between 0 and 1 to determine the alternative with the maximum weighted average. The higher the value of α, the more optimistic is the decision maker.

hypergeometric distribution A distribution of probabilities of the occurrence of x items in a sample of n when there are A of that same item in a population of N.

hypothesis A tentative explanation of a principle operating in nature.

hypothesis testing A process of testing hypotheses about parameters by setting up null and alternative hypotheses, gathering sample data, computing statistics from the samples, and using statistical techniques to reach conclusions about the hypotheses.

I

independent events Events such that the occurrence or nonoccurrence of one has no effect on the occurrence of the others.

independent samples Two or more samples in which the selected items are related only by chance.

independent variable In regression analysis, the predictor variable.

index number A ratio, often expressed as a percentage, of a measure taken during one time frame to that same measure taken during another time frame, usually denoted as the base period.

indicator variable Another name for a dummy or qualitative variable; usually coded as 0 or 1 and represents whether or not a given item or person possesses a certain characteristic.

inferential statistics Statistics that have been gathered from a sample and used to reach conclusions about the population from which the sample was taken.

in-process quality control A quality control method in which product attributes are measured at various intervals throughout the manufacturing process.

interaction When the effects of one treatment in an experimental design vary according to the levels of treatment of the other effect(s).

interquartile range The range of values between the first and the third quartile.

intersection The portion of the population that contains elements that lie in both or all groups of interest.

interval estimate A range of values within which it is estimated with some confidence the population parameter lies.

interval level data Next to highest level of data. These data have all the properties of ordinal level data, but in addition, intervals between consecutive numbers have meaning.

irregular fluctuations Unexplained or error variation within time-series data.

Ishikawa diagram A tool developed by Kaoru Ishikawa as a way to display possible causes of a quality problem and the interrelationships of the causes; also called a fishbone diagram or a cause-and-effect diagram.

J

joint probability The probability of the intersection occurring, or the probability of two or more events happening at once.

judgment sampling A nonrandom sampling technique in which items selected for the sample are chosen by the judgment of the researcher.

just-in-time inventory system An inventory system in which little or no extra raw materials or parts for production are stored.

K

Kruskal-Wallis test The nonparametric alternative to one-way analysis of variance; used to test whether three or more samples come from the same or different populations.

kurtosis The amount of peakedness of a distribution.

L

lambda (λ) Denotes the long-run average of a Poisson distribution.

Laspeyres price index A type of weighted aggregate price index in which the quantity values used in the calculations are from the base year.

lean manufacturing A quality-management philosophy that focuses on the reduction of wastes and the elimination of unnecessary steps in an operation or process.

least squares analysis The process by which a regression model is developed based on calculus techniques that attempt to produce a minimum sum of the squared error values.

leptokurtic Distributions that are high and thin.

level of significance The probability of committing a Type I error; also known as alpha.

levels The subcategories of the independent variable used by the researcher in the experimental design; also called classifications.

lower control limit (LCL) The bottom-end line of a control chart, usually situated approximately three standard deviations of the statistic below the centerline; data points below this line indicate quality control problems.

M

Mann-Whitney *U* test A nonparametric counterpart of the *t* test used to compare the means of two independent populations.

manufacturing quality A view of quality in which the emphasis is on the manufacturer's ability to target consistently the requirements for the product with little variability.

marginal probability A probability computed by dividing a subtotal of the population by the total of the population.

matched-pairs test A *t* test to test the differences in two related or matched samples; sometimes called the *t* test for related measures or the correlated *t* test.

maximax criterion An optimistic approach to decision making under uncertainty in which the decision alternative is chosen according to which alternative produces the maximum overall payoff of the maximum payoffs from each alternative.

maximin criterion A pessimistic approach to decision making under uncertainty in which the decision alternative is chosen according to which alternative produces the maximum overall payoff of the minimum payoffs from each alternative.

mean The long-run average of occurrences; also called the expected value.

mean absolute deviation (MAD) The average of the absolute values of the deviations around the mean for a set of numbers.

mean square error (MSE) The average of all errors squared of a forecast for a group of data.

measures of central tendency One type of measure that is used to yield information about the center of a group of numbers.

measures of shape Tools that can be used to describe the shape of a distribution of data.

measures of variability Statistics that describe the spread or dispersion of a set of data.

median The middle value in an ordered array of numbers.

mesokurtic Distributions that are normal in shape—that is, not too high or too flat.

metric data Interval and ratio level data; also called quantitative data.

minimax regret A decision-making strategy in which the decision maker determines the lost opportunity for each decision alternative and selects the decision alternative with the minimum of lost opportunity or regret.

***mn* counting rule** A rule used in probability to count the number of ways two operations can occur if the first operation has *m* possibilities and the second operation has *n* possibilities.

mode The most frequently occurring value in a set of data.

moving average When an average of data from previous time periods is used to forecast the value for ensuing time periods and this average is modified at each new time period by including more recent values not in the previous average and dropping out values from the more distant time periods that were in the average. It is continually updated at each new time period.

multicollinearity A problematic condition that occurs when two or more of the independent variables of a multiple regression model are highly correlated.

multimodal Data sets that contain more than two modes.

multiple comparisons Statistical techniques used to compare pairs of treatment means when the analysis of variance yields an overall significant difference in the treatment means.

multiple regression Regression analysis with one dependent variable and two or more independent variables or at least one nonlinear independent variable.

mutually exclusive events Events such that the occurrence of one precludes the occurrence of the other.

N

naive forecasting models Simple models in which it is assumed that the more recent time periods of data represent the best predictions or forecasts for future outcomes.

nominal level data The lowest level of data measurement; used only to classify or categorize.

nonlinear regression model Multiple regression models in which the models are nonlinear, such as polynomial models, logarithmic models, and exponential models.

nonmetric data Nominal and ordinal level data; also called qualitative data.

nonparametric statistics A class of statistical techniques that make few assumptions about the population and are particularly applicable to nominal and ordinal level data.

nonrandom sampling Sampling in which not every unit of the population has the same probability of being selected into the sample.

nonrandom sampling techniques Sampling techniques used to select elements from the population by any mechanism that does not involve a random selection process.

nonrejection region Any portion of a distribution that is not in the rejection region. If the observed statistic falls in this region, the decision is to fail to reject the null hypothesis.

nonsampling errors All errors other than sampling errors.

normal distribution A widely known and much-used continuous distribution that fits the measurements of many human characteristics and many machine-produced items.

null hypothesis The hypothesis that assumes the status quo—that the old theory, method, or standard is still true; the complement of the alternative hypothesis.

O

observed significance level Another name for the *p*-value method of testing hypotheses.

observed value A statistic computed from data gathered in an experiment that is used in the determination of whether or not to reject the null hypothesis.

ogive A cumulative frequency polygon; plotted by graphing a dot at each class endpoint for the cumulative or decumulative frequency value and connecting the dots.

one-tailed test A statistical test wherein the researcher is interested only in testing one side of the distribution.

one-way analysis of variance The process used to analyze a completely randomized experimental design. This process involves computing a ratio of the variance between treatment levels of the independent variable to the error variance. This ratio is an *F* value, which is then used to determine whether there are any significant differences between the means of the treatment levels.

operating-characteristic (OC) curve In hypothesis testing, a graph of Type II error probabilities for various possible values of an alternative hypotheses.

opportunity loss table A decision table constructed by subtracting all payoffs for a given state of nature from the maximum payoff for that state of nature and doing this for all states of nature; displays the lost opportunities or regret that would occur for a given decision alternative if that particular state of nature occurred.

ordinal level data Next-higher level of data from nominal level data; can be used to order or rank items, objects, or people.

outliers Data points that lie apart from the rest of the points.

P

***p* chart** A quality control chart for attribute compliance that graphs the proportion of sample items in noncompliance with specifications for multiple samples.

***p*-value method** A method of testing hypotheses in which there is no preset level of α. The probability of getting a test statistic at least as extreme as the observed test statistic is computed under the assumption that the null hypothesis is true. This probability is called the *p*-value, and it is the smallest value of α for which the null hypothesis can be rejected.

Paasche price index A type of weighted aggregate price index in which the quantity values used in the calculations are from the year of interest.

parameter A descriptive measure of the population.

parametric statistics A class of statistical techniques that contain assumptions about the population and that are used only with interval and ratio level data.

Pareto analysis A quantitative tallying of the number and types of defects that occur with a product or service, often recorded in a Pareto chart.

Pareto chart A vertical bar chart in which the number and types of defects for a product or service are graphed in order of magnitude from greatest to least.

partial regression coefficient The coefficient of an independent variable in a multiple regression model that represents the increase that will occur in the value of the dependent variable from a one-unit increase in the independent variable if all other variables are held constant.

payoff table A matrix that displays the decision alternatives, the states of nature, and the payoffs for a particular decision-making problem; also called a decision table.

payoffs The benefits or rewards that result from selecting a particular decision alternative.

percentiles Measures of central tendency that divide a group of data into 100 parts.

pie chart A circular depiction of data where the area of the whole pie represents 100% of the data being studied and slices represent a percentage breakdown of the sublevels.

platykurtic Distributions that are flat and spread out.

point estimate An estimate of a population parameter constructed from a statistic taken from a sample.

Poisson distribution A discrete distribution that is constructed from the probability of occurrence of rare events over an interval; focuses only on the number of discrete occurrences over some interval or continuum.

poka-yoke Means "mistake proofing" and uses devices, methods, or inspections in order to avoid machine error or simple human error.

population A collection of persons, objects, or items of interest.

post hoc After the experiment; pairwise comparisons made by the researcher *after* determining that there is a significant overall *F* value from ANOVA; also called a *posteriori*.

power The probability of rejecting a false null hypothesis.

power curve A graph that plots the power values against various values of the alternative hypothesis.

prediction interval A range of values used in regression analysis to estimate a single value of *y* for a given value of *x*.

probabilistic model A model that includes an error term that allows for various values of output to occur for a given value of input.

probability matrix A two-dimensional table that displays the marginal and intersection probabilities of a given problem.

process A series of actions, changes, or functions that bring about a result.

product quality A view of quality in which quality is measurable in the product based on the fact that there are perceived differences in products and quality products possess more attributes.

proportionate stratified random sampling A type of stratified random sampling in which the proportions of the items selected for the sample from the strata reflect the proportions of the strata in the population.

Q

quadratic regression model A multiple regression model in which the predictors are a variable and the square of the variable.

qualitative variable Another name for a dummy or indicator variable; represents whether or not a given item or person possesses a certain characteristic and is usually coded as 0 or 1.

quality When a product delivers what is stipulated in its specifications.

quality circle A small group of workers consisting of supervisors and six to 10 employees who meet frequently and regularly to consider quality issues in their department or area of the business.

quality control The collection of strategies, techniques, and actions taken by an organization to ensure the production of quality products.

quartiles Measures of central tendency that divide a group of data into four subgroups or parts.

quota sampling A nonrandom sampling technique in which the population is stratified on some characteristic and then elements selected for the sample are chosen by nonrandom processes.

R

***R* chart** A plot of sample ranges used in quality control.

R^2 The coefficient of multiple determination; a value that ranges from 0 to 1 and represents the proportion of the dependent variable in a multiple regression model that is accounted for by the independent variables.

random sampling Sampling in which every unit of the population has the same probability of being selected for the sample.

random variable A variable that contains the outcomes of a chance experiment.

randomized block design An experimental design in which there is one independent variable of interest and a second variable, known as a blocking variable, that is used to control for confounding or concomitant variables.

range The difference between the largest and the smallest values in a set of numbers.

ratio level data Highest level of data measurement; contains the same properties as interval level data, with the additional property that zero has meaning and represents the absence of the phenomenon being measured.

rectangular distribution A relatively simple continuous distribution in which the same height is obtained over a range of values; also referred to as the uniform distribution.

reengineering A radical approach to total quality management in which the core business processes of a company is redesigned.

regression analysis The process of constructing a mathematical model or function that can be used to predict or determine one variable by any other variable.

rejection region If a computed statistic lies in this portion of a distribution, the null hypothesis will be rejected.

related measures Another name for matched pairs or paired data in which measurements are taken from pairs of items or persons matched on some characteristic or from a before-and-after design and then separated into different samples.

relative frequency The proportion of the total frequencies that fall into any given class interval in a frequency distribution.

relative frequency of occurrence Assigning probability based on cumulated historical data.

repeated measures design A randomized block design in which each block level is an individual item or person, and that person or item is measured across all treatments.

research hypothesis A statement of what the researcher believes will be the outcome of an experiment or a study.

residual The difference between the actual y value and the y value predicted by the regression model; the error of the regression model in predicting each value of the dependent variable.

residual plot A type of graph in which the residuals for a particular regression model are plotted along with their associated values of x.

response plane A plane fit in a three-dimensional space and that represents the response surface defined by a multiple regression model with two independent first-order variables.

response surface The surface defined by a multiple regression model.

response variable The dependent variable in a multiple regression model; the variable that the researcher is trying to predict.

risk avoider A decision maker who avoids risk whenever possible and is willing to drop out of a game when given the chance even when the payoff is less than the expected monetary value.

risk taker A decision maker who enjoys taking risk and will not drop out of a game unless the payoff is more than the expected monetary value.

robust Describes a statistical technique that is relatively insensitive to minor violations in one or more of its underlying assumptions.

runs test A nonparametric test of randomness used to determine whether the order or sequence of observations in a sample is random.

S

sample A portion of the whole.

sample proportion The quotient of the frequency at which a given characteristic occurs in a sample and the number of items in the sample.

sample-size estimation An estimate of the size of sample necessary to fulfill the requirements of a particular level of confidence and to be within a specified amount of error.

sample space A complete roster or listing of all elementary events for an experiment.

sampling error Error that occurs when the sample is not representative of the population.

scatter plot (chart) A plot or graph of the pairs of data from a simple regression analysis.

search procedures Processes whereby more than one multiple regression model is developed for a given database, and the models are compared and sorted by different criteria, depending on the given procedure.

seasonal effects Patterns of data behavior that occur in periods of time of less than 1 year, often measured by the month.

serial correlation A problem that arises in regression analysis when the error terms of a regression model are correlated due to time-series data; also called autocorrelation.

set notation The use of braces to group numbers that have some specified characteristic.

simple average The arithmetic mean or average for the values of a given number of time periods of data.

simple average model A forecasting averaging model in which the forecast for the next time period is the average of values for a given number of previous time periods.

simple index number A number determined by computing the ratio of a quantity, price, or cost for a particular year of interest to the quantity price or cost of a base year, expressed as a percentage.

simple random sampling The most elementary of the random sampling techniques; involves numbering each item in the population and using a list or roster of random numbers to select items for the sample.

simple regression Bivariate, linear regression.

Six Sigma A total quality-management approach that measures the capability of a process to perform defect-free work, where a defect is defined as anything that results in customer dissatisfaction.

skewness The lack of symmetry of a distribution of values.

smoothing techniques Forecasting techniques that produce forecasts based on leveling out the irregular fluctuation effects in time-series data.

snowball sampling A nonrandom sampling technique in which survey subjects who fit a desired profile are selected based on referral from other survey respondents who also fit the desired profile.

Spearman's rank correlation A measure of the correlation of two variables; used when only ordinal level or ranked data are available.

standard deviation The square root of the variance.

standard error of the estimate (s_e) A standard deviation of the error of a regression model.

standard error of the mean The standard deviation of the distribution of sample means.

standard error of the proportion The standard deviation of the distribution of sample proportions.

standardized normal distribution z distribution; a distribution of z scores produced for values from a normal distribution with a mean of 0 and a standard deviation of 1.

states of nature The occurrences of nature that can happen after a decision has been made that can affect the outcome of the decision and over which the decision maker has little or no control.

stationary Time-series data that contain no trend, cyclical, or seasonal effects.

statistic A descriptive measure of a sample.

statistical hypothesis A formal hypothesis structure set up with a null and an alternative hypothesis to scientifically test research hypotheses.

statistics A science dealing with the collection, analysis, interpretation, and presentation of numerical data.

stem-and-leaf plot A plot of numbers constructed by separating each number into two groups, a stem and a leaf. The leftmost digits are the stems and the rightmost digits are the leaves.

stepwise regression A step-by-step multiple regression search procedure that begins by developing a regression model with a single predictor variable and adds and deletes predictors one step at a time, examining the fit of the model at each step until there are no more significant predictors remaining outside the model.

stratified random sampling A type of random sampling in which the population is divided into various nonoverlapping strata and then items are randomly selected into the sample from each stratum.

subjective probability A probability assigned based on the intuition or reasoning of the person determining the probability.

substantive result Occurs when the outcome of a statistical study produces results that are important to the decision maker.

sum of squares of error (SSE) The sum of the residuals squared for a regression model.

sum of squares of x The sum of the squared deviations about the mean of a set of values.

systematic sampling A random sampling technique in which every kth item or person is selected from the population.

T

***t* distribution** A distribution that describes the sample data when the standard deviation is unknown and the population is normally distributed.

***t* value** The computed value of t used to reach statistical conclusions regarding the null hypothesis in small-sample analysis.

team building When a group of employees are organized as an entity to undertake management tasks and perform other functions such as organizing, developing, and overseeing projects.

time-series data Data gathered on a given characteristic over a period of time at regular intervals.

total quality management (TQM) A program that occurs when all members of an organization are involved in improving quality; all goals and objectives of the organization come under the purview of quality control and are measured in quality terms.

transcendent quality A view of quality that implies that a product has an innate excellence, uncompromising standards, and high achievement.

treatment variable The independent variable of an experimental design that the researcher either controls or modifies.

trend Long-run general direction of a business climate over a period of several years.

Tukey-Kramer procedure A modification of the Tukey HSD multiple comparison procedure; used when there are unequal sample sizes.

Tukey's four-quadrant approach A graphical method using the four quadrants for determining which expressions of Tukey's ladder of transformations to use.

Tukey's honestly significant difference (HSD) test In analysis of variance, a technique used for pairwise *a posteriori* multiple comparisons to determine if there are significant differences between the means of any pair of treatment levels in an experimental design. This test requires equal sample sizes and uses a q value along with the mean square error in its computation.

Tukey's ladder of transformations A process used for determining ways to recode data in multiple regression analysis to achieve potential improvement in the predictability of the model.

two-stage sampling Cluster sampling done in two stages: A first round of samples is taken and then a second round is taken from within the first samples.

two-tailed test A statistical test wherein the researcher is interested in testing both sides of the distribution.

two-way analysis of variance (two-way ANOVA) The process used to statistically test the effects of variables in factorial designs with two independent variables.

Type I error An error committed by rejecting a true null hypothesis.

Type II error An error committed by failing to reject a false null hypothesis.

U

ungrouped data Raw data, or data that have not been summarized in any way.

uniform distribution A relatively simple continuous distribution in which the same height is obtained over a range of values; also called the rectangular distribution.

union A new set of elements formed by combining the elements of two or more other sets.

union probability The probability of one event occurring or the other event occurring or both occurring.

unweighted aggregate price index number The ratio of the sum of the prices of a market basket of items for a particular year to the sum of the prices of those same items in a base year, expressed as a percentage.

upper control limit (UCL) The top-end line of a control chart, usually situated approximately three standard deviations of the statistic above the centerline; data points above this line indicate quality-control problems.

user quality A view of quality in which the quality of the product is determined by the user.

utility The degree of pleasure or displeasure a decision maker has in being involved in the outcome selection process given the risks and opportunities available.

V

value quality A view of quality having to do with price and costs and whether the consumer got his or her money's worth.

variance The average of the squared deviations about the arithmetic mean for a set of numbers.

variance inflation factor A statistic computed using the R^2 value of a regression model developed by predicting one independent variable of a regression analysis by other independent variables; used to determine whether there is multicollinearity among the variables.

W

weighted aggregate price index number A price index computed by multiplying quantity weights and item prices and summing the products to determine a market basket's worth in a given year and then determining the ratio of the market basket's worth in the year of interest to the same value computed for a base year, expressed as a percentage.

weighted moving average A moving average in which different weights are applied to the data values from different time periods.

Wilcoxon matched-pairs signed rank test A nonparametric alternative to the *t* test for two related or dependent samples.

X

$\bar{x}$ chart A quality control chart for measurements that graphs the sample means computed for a series of small random samples over a period of time.

Z

***z* distribution** A distribution of *z* scores; a normal distribution with a mean of 0 and a standard deviation of 1.

***z* score** The number of standard deviations a value (x) is above or below the mean of a set of numbers when the data are normally distributed.

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